



The next example shows how we can simplify expressions involving exponents by using repeated multiplication.

**Example 2** Expand and multiply:

a.  $3^2$

b.  $4^2$

c.  $3^3$

d.  $2^4$

**Solution**

a.  $3^2 = 3 \cdot 3 = 9$

b.  $4^2 = 4 \cdot 4 = 16$

c.  $3^3 = 3 \cdot 3 \cdot 3 = 9 \cdot 3 = 27$

d.  $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 4 \cdot 4 = 16$

### Using Technology Calculators

Here is how we use a calculator to evaluate exponents, as we did in Example 2d:

**Scientific Calculator**  $2 \boxed{x^y} 4 \boxed{=}$

**Graphing Calculator**  $2 \boxed{\wedge} 4 \boxed{\text{ENT}}$  or  $2 \boxed{x^y} 4 \boxed{\text{ENT}}$   
(depending on the calculator)

Finally, we should consider what happens when the numbers 0 and 1 are used as exponents. First of all, any number raised to the first power is itself. That is, if we let the letter  $a$  represent any number, then

$$a^1 = a$$

To take care of the cases when 0 is used as an exponent, we must use the following definition:

Any number other than 0 raised to the 0 power is 1. That is, if  $a$  represents any nonzero number, then it is always true that

$$a^0 = 1$$

**Example 3** Simplify:

a.  $5^1$

b.  $9^1$

c.  $4^0$

d.  $8^0$

**Solution**

a.  $5^1 = 5$

b.  $9^1 = 9$

c.  $4^0 = 1$

d.  $8^0 = 1$



## Order of Operations

The symbols we use to specify operations,  $+$ ,  $-$ ,  $\cdot$ ,  $\div$ , along with the symbols we use for grouping,  $( )$  and  $[ ]$ , serve the same purpose in mathematics as punctuation marks in English. They may be called the punctuation marks of mathematics.

Consider the following sentence:

Bob said John is tall.

It can have two different meanings, depending on how we punctuate it:

1. “Bob,” said John, “is tall.”
2. Bob said, “John is tall.”

Without the punctuation marks we don’t know which meaning the sentence has.

Now, consider the following mathematical expression:

$$4 + 5 \cdot 2$$

What should we do? Should we add 4 and 5 first, or should we multiply 5 and 2 first? There seem to be two different answers. In mathematics we want to avoid situations in which two different results are possible. Therefore we follow the rule for order of operations.

### Order of Operations

When evaluating mathematical expressions, we will perform the operations in the following order:

1. If the expression contains grouping symbols, such as parentheses  $( )$ , brackets  $[ ]$ , or a fraction bar, then we perform the operations inside the grouping symbols, or above and below the fraction bar, first.
2. Then we evaluate, or simplify, any numbers with exponents.
3. Then we do all multiplications and divisions in order, starting at the left and moving right.
4. Finally, we do all additions and subtractions, from left to right.

According to our rule, the expression  $4 + 5 \cdot 2$  would have to be evaluated by multiplying 5 and 2 first, and then adding 4. The correct answer—and the only answer—to this problem is 14.

$$\begin{aligned} 4 + 5 \cdot 2 &= 4 + 10 && \text{Multiply first} \\ &= 14 && \text{Then add} \end{aligned}$$

Here are some more examples that illustrate how we apply the rule for order of operations to simplify (or evaluate) expressions.

**Example 4** Simplify:  $4 \cdot 8 - 2 \cdot 6$

**Solution** We multiply first and then subtract:

$$\begin{aligned} 4 \cdot 8 - 2 \cdot 6 &= 32 - 12 && \text{Multiply first} \\ &= 20 && \text{Then subtract} \end{aligned}$$

**Example 5** Simplify:  $5 + 2(7 - 1)$

**Solution** According to the rule for the order of operations, we must do what is inside the parentheses first:

$$\begin{aligned} 5 + 2(7 - 1) &= 5 + 2(6) && \text{Inside parentheses first} \\ &= 5 + 12 && \text{Then multiply} \\ &= 17 && \text{Then add} \end{aligned}$$

**Example 6** Simplify:  $9 \cdot 2^3 + 36 \div 3^2 - 8$

**Solution**

$$\begin{aligned} 9 \cdot 2^3 + 36 \div 3^2 - 8 &= 9 \cdot 8 + 36 \div 9 - 8 && \text{Exponents first} \\ &= 72 + 4 - 8 && \text{Then multiply and divide, left to right} \\ &= 76 - 8 && \text{Add and subtract, left to right} \\ &= 68 \end{aligned}$$

### Using Technology Calculators

Here is how we use a calculator to work the problem shown in Example 5:

**Scientific Calculator**  $5 \boxed{+} 2 \boxed{\times} (\boxed{7} \boxed{-} \boxed{1}) \boxed{=}$

**Graphing Calculator**  $5 \boxed{+} 2 \boxed{(} \boxed{7} \boxed{-} \boxed{1} \boxed{)} \boxed{=}$

Example 6 on a calculator looks like this:

**Scientific Calculator**  $9 \boxed{\times} 2 \boxed{x^y} 3 \boxed{+} 36 \boxed{\div} 3 \boxed{x^y} 2 \boxed{-} 8 \boxed{=}$

**Graphing Calculator**  $9 \boxed{\times} 2 \boxed{\wedge} 3 \boxed{+} 36 \boxed{\div} 3 \boxed{\wedge} 2 \boxed{-} 8 \boxed{=}$

**Example 7** Simplify:  $3 + 2[10 - 3(5 - 2)]$

**Solution** The brackets,  $[ ]$ , are used in the same way as parentheses. In a case like this we move to the innermost grouping symbols first and begin simplifying:

$$\begin{aligned} 3 + 2[10 - 3(5 - 2)] &= 3 + 2[10 - 3(3)] \\ &= 3 + 2[10 - 9] \\ &= 3 + 2[1] \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

Table 1 lists some English expressions and their corresponding mathematical expressions written in symbols.

In English	Mathematical Equivalent
5 times the sum of 3 and 8	$5(3 + 8)$
Twice the difference of 4 and 3	$2(4 - 3)$
6 added to 7 times the sum of 5 and 6	$6 + 7(5 + 6)$
The sum of 4 times 5 and 8 times 9	$4 \cdot 5 + 8 \cdot 9$
3 subtracted from the quotient of 10 and 2	$10 \div 2 - 3$

Table 1



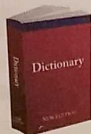
**Example 8** Translate the English expression “5 added to 3 times the difference of 11 and 7” into an equivalent mathematical expression written in symbols. Then simplify using the order of operations.

**Solution**

$$\begin{array}{c}
 5 \quad \text{added to } 3 \text{ times the difference of } 11 \text{ and } 7 \\
 \swarrow \quad \downarrow \quad \searrow \quad \swarrow \\
 5 + 3(11 - 7) = 5 + 3(4) \\
 = 5 + 12 = 17
 \end{array}$$

### Descriptive Statistics Average

Next we turn our attention to averages. If we go online to the Merriam-Webster dictionary at [www.m-w.com](http://www.m-w.com), we find the following definition for the word average when it is used as a noun:



**av · er · age** *noun*: a single value (as a mean, mode, or median) that summarizes or represents the general significance of a set of unequal values ...

In everyday language, the word *average* can refer to the mean, the median, or the mode. The mean is probably the most common average.

## Mean

### Mean

To find the **mean** for a set of numbers, we add all the numbers and then divide the sum by the number of numbers in the set. The mean is sometimes called the **arithmetic mean**.



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**Example 9** A firefighter in a large southern California city earned the following salaries for his first five years on the job. Find the mean of these salaries.

\$59,687   \$60,880   \$62,098   \$63,651   \$65,879

**Solution** We add the five numbers and then divide by 5, the number of numbers in the set.

$$\text{Mean} = \frac{59,687 + 60,880 + 62,098 + 63,651 + 65,879}{5} = \frac{312,195}{5} = 62,439$$

The firefighter’s mean salary for the first five years was \$62,439 per year.

## Median

The table below shows the hourly wages for some occupations in the U.S. in May 2011.

All Occupations	\$16.57
Waiters/Waitresses	\$8.93
School Bus Driver	\$13.51
Clergy	\$21.22
Firefighter	\$21.84
Social Worker	\$25.91
Veterinarians	\$39.86
Lawyer	\$54.48

Source: U.S. Bureau of Labor Statistics  
(all wages are median figures for 2011)

Table 2

If you look at the type at the bottom of Table 2, you can see that the numbers are the *median* figures for 2011. The median for a set of numbers is the number such that half of the numbers in the set are above it and half are below it. Here is the exact definition.

### Median

To find the **median** for a set of numbers, we write the numbers in order from smallest to largest. If there is an odd number of numbers, the median is the middle number. If there is an even number of numbers, then the median is the mean of the two numbers in the middle.

**Example 10** Find the median of the numbers given in Example 8.

**Solution** The numbers in Example 9, written from smallest to largest, are shown below. Because there is an odd number of numbers in the set, the median is the middle number.

59,687   60,880   62,098   63,651   65,879

↑  
median

The firefighter's median salary for his first five years on the job is \$62,098.

**Example 11** Four firefighters in a small town earned the following first-year salaries.

\$42,926   \$40,662   \$41,474   \$40,050

Find the mean and the median for the four salaries.

**Solution** To find the mean, we add the four numbers and then divide by 4:

$$\frac{42,926 + 40,662 + 41,474 + 40,050}{4} = \frac{165,112}{4} = 41,278$$



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To find the median, we write the numbers in order from smallest to largest. Then, because there is an even number of numbers, we average the middle two numbers to obtain the median.

$$\begin{array}{ccccccc}
 40,050 & 40,662 & 41,474 & 42,926 & & & \\
 & \underbrace{\hspace{1.5cm}} & & & & & \\
 & \text{median} & & & & & \\
 & \downarrow & & & & & \\
 & \frac{40,662 + 41,474}{2} & = & 41,068 & & & 
 \end{array}$$

The mean is \$41,278, and the median is \$41,068.

## Mode

The mode is best used when we are looking for the most common eye color in a group of people, the most popular breed of dog in the United States, or the movie that was seen the most often. When we have a set of numbers in which one number occurs more often than the rest, that number is the mode.

### Mode

The **mode** for a set of numbers is the number that occurs most frequently. If all the numbers in the set occur the same number of times, there is no mode.

**Example 12** A math class with 18 students had the grades shown below on their first test. Find the mean, the median, and the mode.

77 87 100 65 79 87  
 79 85 87 95 56 87  
 56 75 79 93 97 92

**Solution** To find the mean, we add all the numbers and divide by 18:

mean =

$$\begin{aligned}
 & \frac{77 + 87 + 100 + 65 + 79 + 87 + 79 + 85 + 87 + 95 + 56 + 87 + 56 + 75 + 79 + 93 + 97 + 92}{18} \\
 & = \frac{1,476}{18} = 82
 \end{aligned}$$

To find the median, we must put the test scores in order from smallest to largest; then, because there is an even number of test scores, we must find the mean of the middle two scores.

56 56 65 75 77 79 79 79 85 87 87 87 87 92 93 95 97 100

$$\text{Median} = \frac{85 + 87}{2} = 86$$

The mode is the most frequently occurring score. Because 87 occurs 4 times, and no other scores occur that many times, 87 is the mode.

The mean is 82, the median is 86, and the mode is 87.

## More Vocabulary

When we used the word *average* for the first time in this section, we used it as a noun. It can also be used as an adjective and a verb. Below is the definition of the word *average* when it is used as a verb.

### Average

**av · er · age** *verb*: to find the arithmetic mean of (a series of unequal quantities) ...

In everyday language, if you are asked for, or given, the *average* of a set of numbers, the word *average* can represent the mean, the median, or the mode. When used in this way, the word *average* is a noun. However, if you are asked to *average* a set of numbers, then the word *average* is a verb, and you are being asked to find the mean of the numbers.

Before we leave this section, there is one more statistic we need. It is called the *range*, and it is used to give us an idea of how spread out the numbers in our samples are. Here is the definition.

### Range

The **range** for a set of numbers is the difference between the largest number and the smallest number in the sample.

If we look back to Example 9, we find the range of salaries is

$$\text{Range} = 65,879 - 59,687 = \$6,192$$

Likewise, the range of test scores in Example 12 is

$$\text{Range} = 100 - 56 = 44$$

### Getting Ready for Class

*After reading through the preceding section, respond in your own words and in complete sentences.*

- A. In the expression  $5^3$ , which number is the base?  
 B. Give a written description of the process you would use to simplify the expression below.

$$3 + 4(5 + 6)$$

- C. What is the first step in simplifying the expression below?

$$8 + 6 \div 3 - 1$$

- D. What number must we use for  $x$ , if the mean of 6, 8, and  $x$  is to be 8?



## Problem Set R.7

For each of the following expressions, name the base and the exponent.

1.  $4^5$       2.  $5^4$       3.  $3^6$       4.  $6^3$       5.  $8^2$   
6.  $2^8$       7.  $9^1$       8.  $1^9$       9.  $4^0$       10.  $0^4$

Use the definition of exponents as indicating repeated multiplication to simplify each of the following expressions.

11.  $6^2$     12.  $7^2$     13.  $2^3$     14.  $2^4$     15.  $1^4$     16.  $5^1$     17.  $9^0$     18.  $27^0$   
19.  $9^2$     20.  $8^2$     21.  $10^1$     22.  $8^1$     23.  $12^1$     24.  $16^0$     25.  $45^0$     26.  $3^4$

Use the rule for the order of operations to simplify each expression.

27.  $16 - 8 + 4$       28.  $16 - 4 + 8$       29.  $20 \div 2 \cdot 10$   
30.  $40 \div 4 \cdot 5$       31.  $20 - 4 \cdot 4$       32.  $30 - 10 \cdot 2$   
33.  $3 + 5 \cdot 8$       34.  $7 + 4 \cdot 9$       35.  $3 \cdot 6 - 2$   
36.  $5 \cdot 1 + 6$       37.  $6 \cdot 2 + 9 \cdot 8$       38.  $4 \cdot 5 + 9 \cdot 7$   
39.  $4 \cdot 5 - 3 \cdot 2$       40.  $5 \cdot 6 - 4 \cdot 3$       41.  $5^2 + 7^2$   
42.  $4^2 + 9^2$       43.  $480 + 12(32)^2$       44.  $360 + 14(27)^2$   
45.  $3 \cdot 2^3 + 5 \cdot 4^2$       46.  $4 \cdot 3^2 + 5 \cdot 2^3$       47.  $8 \cdot 10^2 - 6 \cdot 4^3$   
48.  $5 \cdot 11^2 - 3 \cdot 2^3$       49.  $2(3 + 6 \cdot 5)$       50.  $8(1 + 4 \cdot 2)$   
51.  $19 + 50 \div 5^2$       52.  $9 + 8 \div 2^2$       53.  $9 - 2(4 - 3)$   
54.  $15 - 6(9 - 7)$       55.  $4 \cdot 3 + 2(5 - 3)$       56.  $6 \cdot 8 + 3(4 - 1)$   
57.  $4[2(3) + 3(5)]$       58.  $3[2(5) + 3(4)]$       59.  $(7 - 3)(8 + 2)$   
60.  $(9 - 5)(9 + 5)$       61.  $3(9 - 2) + 4(7 - 2)$       62.  $7(4 - 2) - 2(5 - 3)$   
63.  $18 + 12 \div 4 - 3$       64.  $20 + 16 \div 2 - 5$       65.  $4(10^2) + 20 \div 4$   
66.  $3(4^2) + 10 \div 5$       67.  $8 \cdot 2^4 + 25 \div 5 - 3^2$       68.  $5 \cdot 3^4 + 16 \div 8 - 2^2$   
69.  $5 + 2[9 - 2(4 - 1)]$       70.  $6 + 3[8 - 3(1 + 1)]$       71.  $3 + 4[6 + 8(2 - 0)]$   
72.  $2 + 5[9 + 3(4 - 1)]$       73.  $\frac{15 + 5(4)}{17 - 12}$       74.  $\frac{20 + 6(2)}{11 - 7}$

**Paying Attention to Instructions** The following two problems are intended to give you practice reading, and paying attention to, the instructions that accompany the problems you are working.

75. a. Find the sum of 3, 5, 7, 9, and 11.  
b. Find the mean of 3, 5, 7, 9, and 11.  
c. Find the range of 3, 5, 7, 9, and 11.
76. a. Find the sum of 12, 14, 16, and 18.  
b. Find the mean of 12, 14, 16, and 18.  
c. Find the median of 12, 14, 16, and 18.



Translate each English expression into an equivalent mathematical expression written in symbols. Then simplify.

77. Twice the sum of 10 and 3                      78. 5 times the difference of 12 and 6
79. 4 added to 3 times the sum of 3 and 4
80. 25 added to 4 times the difference of 7 and 5
81. 9 subtracted from the quotient of 20 and 2
82. 7 added to the quotient of 6 and 2
83. The sum of 8 times 5 and 5 times 4
84. The difference of 10 times 5 and 6 times 2

Find the mean and the range for each set of numbers.

85. 1, 2, 3, 4, 5      86. 2, 4, 6, 8, 10      87. 1, 3, 9, 11      88. 5, 7, 9, 12, 12

Find the median and the range for each set of numbers

89. 5, 9, 11, 13, 15                                      90. 10, 20, 50, 90, 100
91. 42, 48, 50, 64                                      92. 700, 900, 1,100

Find the mode and the range for each set of numbers.

93. 14, 18, 27, 36, 18, 73                              94. 11, 27, 18, 11, 72, 11

## Applying the Concepts

**Nutrition Labels** Use the three nutrition labels below to work Problems 95–100.

Spaghetti

Nutrition Facts	
Serving Size 2 oz. (56g/1/8 of pkg) dry	
Servings Per Container: 8	
Amount Per Serving	
Calories 210	Calories from fat 10
% Daily Value*	
Total Fat 1g	2%
Saturated Fat 0g	0%
Poly unsaturated Fat 0.5g	
Monounsaturated Fat 0g	
Cholesterol 0mg	0%
Sodium 0mg	0%
Total Carbohydrate 42g	14%
Dietary Fiber 2g	7%
Sugars 3g	
Protein 7g	
Vitamin A 0%	Vitamin C 0%
Calcium 0%	Iron 10%
*Percent Daily Values are based on a 2,000 calorie diet	

Canned Italian Tomatoes

Nutrition Facts	
Serving Size 1/2 cup (121g)	
Servings Per Container: about 3 1/2	
Amount Per Serving	
Calories 25	Calories from fat 0
% Daily Value*	
Total Fat 0g	0%
Saturated Fat 0g	0%
Cholesterol 0mg	0%
Sodium 300mg	12%
Potassium 145mg	4%
Total Carbohydrate 4g	2%
Dietary Fiber 1g	4%
Sugars 4g	
Protein 1g	
Vitamin A 20%	Vitamin C 15%
Calcium 4%	Iron 15%
*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.	

Shredded Romano Cheese

Nutrition Facts	
Serving Size 2 tsp (5g)	
Servings Per Container: 34	
Amount Per Serving	
Calories 20	Calories from fat 10
% Daily Value*	
Total Fat 1.5g	2%
Saturated Fat 1g	5%
Cholesterol 5mg	2%
Sodium 70mg	3%
Total Carbohydrate 0g	0%
Fiber 0g	0%
Sugars 0g	
Protein 2g	
Vitamin A 0%	Vitamin C 0%
Calcium 4%	Iron 0%
*Percent Daily Values (DV) are based on a 2,000 calorie diet	





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Find the total number of calories in each of the following meals.

- |                      |            |                      |            |
|----------------------|------------|----------------------|------------|
| <b>95.</b> Spaghetti | 1 serving  | <b>96.</b> Spaghetti | 1 serving  |
| Tomatoes             | 1 serving  | Tomatoes             | 2 servings |
| Cheese               | 1 serving  | Cheese               | 1 serving  |
| <b>97.</b> Spaghetti | 2 servings | <b>98.</b> Spaghetti | 2 servings |
| Tomatoes             | 1 serving  | Tomatoes             | 1 serving  |
| Cheese               | 1 serving  | Cheese               | 2 servings |

Find the number of calories from fat in each of the following meals.

- |                      |           |                       |            |
|----------------------|-----------|-----------------------|------------|
| <b>99.</b> Spaghetti | 2 serving | <b>100.</b> Spaghetti | 2 serving  |
| Tomatoes             | 1 serving | Tomatoes              | 1 servings |
| Cheese               | 1 serving | Cheese                | 2 serving  |

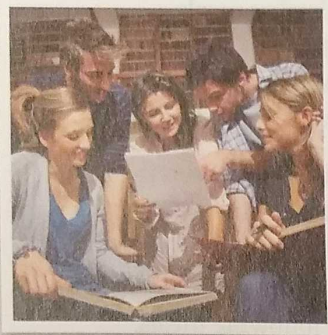
The following table lists the number of calories consumed by eating some popular fast foods. Use the table to work Problems 101 and 102.

Calories in Food	
Food	Calories
McDonald's hamburger	250
Burger King hamburger	240
Jack in the Box hamburger	290
McDonald's Big Mac	550
Burger King Whopper	630
Jack in the Box Jumbo Jack	540

- 101.** Compare the total number of calories in the meal in Problem 95 with the number of calories in a McDonald's Big Mac.
- 102.** Compare the total number of calories in the meal in Problem 98 with the number of calories in a Burger King hamburger.
- 103. Average** If a basketball team has scores of 61, 76, 98, 55, 76, and 102 in their first six games, find
- the mean score
  - the median score
  - the mode of the scores
  - the range of scores
- 104. Home Sales** Below are listed the prices paid for 10 homes that sold during the month of February in a city in Texas.
- |           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|
| \$210,000 | \$139,000 | \$122,000 | \$145,000 | \$120,000 |
| \$540,000 | \$167,000 | \$125,000 | \$125,000 | \$950,000 |
- Find the mean housing price for the month.
  - Find the median housing price for the month.
  - Find the mode of the housing prices for the month.
  - Which measure of "average" best describes the average housing price for the month? Explain your answer.



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- 105. Average Enrollment** The number of students enrolled in a community college during a 5-year period is shown in this table:

Year	Enrollment
2008	6,789
2009	6,970
2010	7,242
2011	6,981
2012	6,423

Find the mean enrollment and the range of enrollments for this 5-year period.

- 106. Car Prices** The following prices were listed for Volkswagen Jettas on the ebay.com car auction site. Use the table to find each of the following:

- the mean car price
- the median car price
- the mode for the car prices
- the range of car prices

Car Prices	
Year	Price
2006	\$10,000
2007	\$14,500
2007	\$10,500
2007	\$11,700
2007	\$15,500
2008	\$10,500
2010	\$18,200
2011	\$19,900