Division with Whole Numbers

R.6

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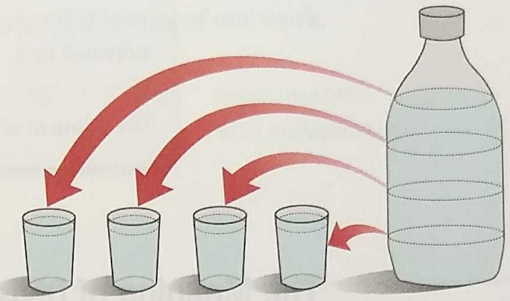
Darlene is planning a party and would like to serve 8-ounce glasses of soda. The glasses will be filled from 32-ounce bottles of soda. In order to know how many bottles of soda to buy, she needs to find out how many of the 8-ounce glasses can be filled by one of the 32-ounce bottles. One way to solve this problem is with division: dividing 32 by 8. A diagram of the problem is shown in Figure 1.



8-ounce glasses 32-ounce bottle

Figure 1

As a division problem:	As a multiplication problem:
$32 \div 8 = 4$	$4 \cdot 8 = 32$

Notation

As was the case with multiplication, there are many ways to indicate division. All the following statements are equivalent. They all mean 10 divided by 5.

$$10 \div 5, \quad \frac{10}{5}, \quad 10/5, \quad 5\overline{)10}$$

The kind of notation we use to write division problems will depend on the situation. We will use the notation $5\overline{)10}$ mostly with the long division problems found in this chapter. The notation $\frac{10}{5}$ will be used in the chapter on fractions and in later chapters. The horizontal line used with the notation $\frac{10}{5}$ is called the *fraction bar.*

Vocabulary

The word *quotient* is used to indicate division. If we say "The quotient of 10 and 5 is 2," then we mean

$$10 \div 5 = 2 \quad \text{or} \quad \frac{10}{5} = 2$$

The 10 is called the *dividend,* and the 5 is called the *divisor.* All the expressions, $10 \div 5$, $\frac{10}{5}$, and 2, are called the *quotient* of 10 and 5.

In English	In Symbols
The quotient of 15 and 3	$15 \div 3$, or $\frac{15}{3}$, or $15/3$
The quotient of 3 and 15	$3 \div 15$, or $\frac{3}{15}$, or $3/15$
The quotient of 8 and n	$8 \div n$, or $\frac{8}{n}$, or $8/n$
x divided by 2	$x \div 2$, or $\frac{x}{2}$, or $x/2$
The quotient of 21 and 3 is 7	$21 \div 3 = 7$, or $\frac{21}{3} = 7$

Table 1

The Meaning of Division

One way to arrive at an answer to a division problem is by thinking in terms of multiplication. For example, if we want to find the quotient of 32 and 8, we may ask, "What do we multiply by 8 to get 32?"

$$32 \div 8 = ? \qquad \text{means} \qquad 8 \cdot ? = 32$$

Because we know from our work with multiplication that $8 \cdot 4 = 32$, it must be true that

$$32 \div 8 = 4$$

Table 2 lists some additional examples.

Division		Multiplication
$18 \div 6 = 3$	because	$6 \cdot 3 = 18$
$32 \div 8 = 4$	because	$8 \cdot 4 = 32$
$10 \div 2 = 5$	because	$2 \cdot 5 = 10$
$72 \div 9 = 8$	because	$9 \cdot 8 = 72$

Table 2

Division by One-Digit Numbers

Consider the following division problem:

$$465 \div 5$$

We can think of this problem as asking the question, "How many fives can we subtract from 465?" To answer the question we begin subtracting multiples of 5. One way to organize this process is shown below:

$$\begin{array}{r} 90 \\ 5\overline{)465} \\ -\ 450 \\ \hline 15 \end{array}$$

← We first guess that there are at least 90 fives in 465

← $90(5) = 450$

← 15 is left after we subtract 90 fives from 465

What we have done so far is subtract 90 fives from 465 and found that 15 is still left. Because there are 3 fives in 15, we continue the process.

$$\begin{array}{r} 3 \\ 90 \\ 5\overline{)465} \\ -\,450 \\ \hline 15 \\ -\,15 \\ \hline 0 \end{array} \quad \begin{array}{l} \leftarrow \text{There are 3 fives in 15} \\ \\ \\ \\ \\ \leftarrow 3 \cdot 5 = 15 \\ \leftarrow \text{The difference is 0} \end{array}$$

The total number of fives we have subtracted from 465 is $90 + 3 = 93$.

We now summarize the results of our work.

$$465 \div 5 = 93$$ which we check with multiplication $\longrightarrow$ $\begin{array}{r} {\scriptstyle 1} \\ 93 \\ \times\ 5 \\ \hline 465 \end{array}$

Notation

The division problem just shown can be shortened by eliminating the zeros in each estimate, since they act simply as placeholders.

The shorthand form for this problem

$$\begin{array}{r} 3 \\ 90 \\ 5\overline{)465} \\ -\,450 \\ \hline 15 \\ -\,15 \\ \hline 0 \end{array}$$ looks like this $$\begin{array}{r} 93 \\ 5\overline{)465} \\ -\,45\downarrow \\ \hline 15 \\ -\,15 \\ \hline 0 \end{array}$$

The arrow indicates that we bring down the 5 after we subtract

The problem shown above on the right is the shortcut form of what is called *long division*. Here is an example showing this shortcut form of long division from start to finish.

VIDEO EXAMPLES

SECTION R.6

Example 1 Divide $595 \div 7$.

Solution Because $7(8) = 56$, our first estimate of the number of sevens that can be subtracted from 595 is 80.

$$\begin{array}{r} 8 \\ 7\overline{)595} \\ -\,56\downarrow \\ \hline 35 \end{array} \quad \begin{array}{l} \leftarrow \text{The 8 is placed above the tens column so we know our first estimate is 80} \\ \\ \leftarrow 8(7) = 56 \\ \leftarrow 59 - 56 = 3\text{; then bring down the 5} \end{array}$$

Since $7(5) = 35$, we have

$$\begin{array}{r} 85 \\ 7\overline{)595} \\ -\,56\downarrow \\ \hline 35 \\ -\,35 \\ \hline 0 \end{array} \quad \begin{array}{l} \leftarrow \text{There are 5 sevens in 35} \\ \\ \\ \\ \leftarrow 5(7) = 35 \\ \leftarrow 35 - 35 = 0 \end{array}$$

Our result is $595 \div 7 = 85$, which we can check with multiplication:

$$\begin{array}{r} {\scriptstyle 3} \\ 85 \\ \times\ 7 \\ \hline 595 \end{array}$$

Division by Two-Digit Numbers

Example 2 Divide: 9,380 ÷ 35

Solution In this case our divisor, 35, is a two-digit number. The process of division is the same. We still want to find the number of thirty-fives we can subtract from 9,380.

```
      2          ← The 2 is placed above the hundreds column
35)9,380
 − 70↓           ← 2(35) = 70
   238           ← 93 − 70 = 23; then bring down the 8
```

We can make a few preliminary calculations to help estimate how many thirty-fives are in 238:

$$5 \times 35 = 175 \qquad 6 \times 35 = 210 \qquad 7 \times 35 = 245$$

Because 210 is the closest to 238 without being larger than 238, we use 6 as our next estimate:

```
      26          ← 6 in the tens column means this estimate is 60
35)9,380
 − 70 |
   238 |
 − 210↓          ← 6(35) = 210
   280           ← 238 − 210; bring down the 0
```

Because 35(8) = 280, we have

```
      268
35)9,380
 − 70
   238
 − 210
   280
 − 280           ← 8(35) = 280
     0           ← 280 − 280 = 0
```

We can check our result with multiplication:

$$\begin{array}{r} 268 \\ \times\ 35 \\ \hline 1{,}340 \\ +\ 8{,}040 \\ \hline 9{,}380 \end{array}$$

Example 3 Divide: 1,872 by 18.

Solution Here is the first step.

```
      1           ← 1 is placed above hundreds column
18)1,872
 − 18            ← Multiply 1(18) to get 18
   0             ← Subtract to get 0
```

The next step is to bring down the 7 and divide again.

$$\begin{array}{r} 10 \\ 18\overline{)1{,}872} \\ -\,18\downarrow \\ \hline 07 \\ -\,0 \\ \hline 7 \end{array}$$

← 0 is placed above tens column. 0 is the largest number we can multiply by 18 and not go over 7

← Multiply 0(18) to get 0

← Subtract to get 7

Here is the complete problem.

$$\begin{array}{r} 104 \\ 18\overline{)1{,}872} \\ -\,18\downarrow \\ \hline 07 \\ -\,0\downarrow \\ \hline 72 \\ -\,72 \\ \hline 0 \end{array}$$

To show our answer is correct, we multiply.

$$18(104) = 1{,}872$$

Division with Remainders

Suppose Darlene was planning to use 6-ounce glasses instead of 8-ounce glasses for her party. To see how many glasses she could fill from the 32-ounce bottle, she would divide 32 by 6. If she did so, she would find that she could fill 5 glasses, but after doing so she would have 2 ounces of soda left in the bottle. A diagram of this problem is shown in Figure 2.

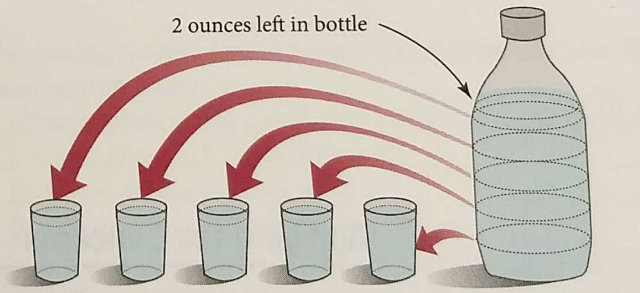


Figure 2

Writing the results in the diagram as a division problem looks like this:

$$\begin{array}{rr} & 5 \\ \text{Divisor} \longrightarrow & 6\overline{)32} \\ & -\,30 \\ \hline & 2 \end{array}$$

← Quotient

← Dividend

← Remainder

Example 4 Divide: $1{,}690 \div 67$

Solution Dividing as we have previously, we get

$$\begin{array}{r} 25 \\ 67\overline{)1{,}690} \\ -\ 1\,34\downarrow \\ \hline 350 \\ -\ 335 \\ \hline 15 \end{array} \quad \longleftarrow \text{15 is left over}$$

We have 15 left, and because 15 is less than 67, no more sixty-sevens can be subtracted. In a situation like this we call 15 the remainder and write

These indicate that the remainder is 15

$$\begin{array}{r} 25\ \text{R}\ 15 \\ 67\overline{)1{,}690} \\ -\ 1\,34\downarrow \\ \hline 350 \\ -\ 335 \\ \hline 15 \end{array} \quad \text{or} \quad \begin{array}{r} 25\frac{15}{67} \\ 67\overline{)1{,}690} \\ -\ 1\,34\downarrow \\ \hline 350 \\ -\ 335 \\ \hline 15 \end{array}$$

Both forms of notation shown above indicate that 15 is the remainder. The notation R 15 is the notation we will use in this chapter. The notation $\frac{15}{67}$ will be useful in the chapter on fractions.

To check a problem like this, we multiply the divisor and the quotient as usual, and then add the remainder to this result:

$$\begin{array}{r} 67 \\ \times\ 25 \\ \hline 335 \\ +\ 1{,}340 \\ \hline 1{,}675 \end{array} \quad \longleftarrow \text{Product of divisor and quotient}$$

$$1{,}675 + 15 = 1{,}690$$

Remainder — Dividend

Calculator Note

Here is how we would work the problem shown in Example 4 on a calculator:

Scientific Calculator:

1690 ÷ 67 =

Graphing Calculator:

1690 ÷ 67 ENT

In both cases the calculator will display 25.223881 (give or take a few digits at the end), which gives the remainder in decimal form. We will discuss decimals later in the book.

Applying the Concepts

Example 5 A family has an annual income of $35,880. How much is their average monthly income?

Solution Because there are 12 months in a year and the yearly (annual) income is $35,880, we want to know what $35,880 divided into 12 equal parts is. Therefore we have

$$\begin{array}{r} 2\,990 \\ 12\overline{)35{,}880} \\ -\ 24\downarrow\downarrow \\ \hline 11\,8\downarrow \\ -\ 10\,8\downarrow \\ \hline 1\,08\downarrow \\ -\ 1\,08 \\ \hline 00 \end{array}$$

Because $35{,}880 \div 12 = 2{,}990$, the monthly income for this family is $2,990.

Note To estimate the answer to Example 5 quickly, we can replace 35,880 with 36,000 and mentally calculate

$$36{,}000 \div 12$$

which gives an estimate of 3,000. Our actual answer, 2,990, is close enough to our estimate to convince us that we have not made a major error in our calculation.

Note Don't confuse $\frac{8}{0}$ with $\frac{0}{8}$. As our rule states, $\frac{8}{0}$ is undefined because we cannot divide by 0. In the second case, however, $\frac{0}{8}$ is defined and $\frac{0}{8} = 0$. In fact, 0 divided by any number (except 0) is equal to 0. Can you use multiplication to see why this is so?

Division by Zero

We cannot divide by 0. That is, we cannot use 0 as a divisor in any division problem. Here's why.

Suppose there was an answer to the problem

$$\frac{8}{0} = ?$$

That would mean that

$$0 \cdot ? = 8$$

But we already know that multiplication by 0 always produces 0. There is no number we can use for the ? to make a true statement out of

$$0 \cdot ? = 8$$

Because this was equivalent to the original division problem

$$\frac{8}{0} = ?$$

we have no number to associate with the expression $\frac{8}{0}$. It is undefined.

Rule
Division by 0 is undefined. Any expression with a divisor of 0 is undefined. We cannot divide by 0.

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Which sentence below describes the problem shown in Example 1?
   1. The quotient of 7 and 595 is 85.
   2. Seven divided by 595 is 85.
   3. The quotient of 595 and 7 is 85.

B. In Example 2, we divide 9,380 by 35 to obtain 268. Suppose we add 35 to 9,380, making it 9,415. What will our answer be if we divide 9,415 by 35?

C. Example 4 shows that 1,690 ÷ 67 gives a quotient of 25 with a remainder of 15. If we were to divide 1,692 by 67, what would the remainder be?

D. Explain why division by 0 is undefined in mathematics.

Problem Set R.6

Write each of the following in symbols.

1. The quotient of 6 and 3.
2. The quotient of 3 and 6.
3. The quotient of 45 and 9.
4. The quotient of 12 and 4.
5. The quotient of r and s.
6. The quotient of s and r.
7. The quotient of 20 and 4 is 5.
8. The quotient of 20 and 5 is 4.

Write a multiplication statement that is equivalent to each of the following division statements.

9. $6 \div 2 = 3$
10. $6 \div 3 = 2$
11. $\frac{36}{9} = 4$
12. $\frac{36}{4} = 9$
13. $\frac{48}{6} = 8$
14. $\frac{35}{7} = 5$
15. $28 \div 7 = 4$
16. $81 \div 9 = 9$

Find each of the following quotients. (Divide.)

17. $25 \div 5$
18. $72 \div 8$
19. $40 \div 5$
20. $12 \div 2$
21. $9 \div 0$
22. $7 \div 1$
23. $360 \div 8$
24. $285 \div 5$
25. $\frac{138}{6}$
26. $\frac{267}{3}$
27. $5\overline{)7{,}650}$
28. $5\overline{)5{,}670}$
29. $5\overline{)6{,}750}$
30. $5\overline{)6{,}570}$
31. $3\overline{)54{,}000}$
32. $3\overline{)50{,}400}$
33. $3\overline{)50{,}040}$
34. $3\overline{)50{,}004}$

Estimating

Work Problems 35 through 38 mentally, without using a calculator.

35. The quotient $876 \div 93$ is closest to which of the following numbers?
    a. 10 b. 100 c. 1,000 d. 10,000

36. The quotient $762 \div 43$ is closest to which of the following numbers?
    a. 2 b. 20 c. 200 d. 2,000

37. The quotient $15{,}893 \div 771$ is closest to which of the following numbers?
    a. 2 b. 20 c. 200 d. 2,000

38. The quotient $24{,}684 \div 523$ is closest to which of the following numbers?
    a. 5 b. 50 c. 500 d. 5,000

Without a calculator give a one-digit estimate for each of the following quotients. That is, for each quotient, mentally estimate the answer using one of the digits 1, 2, 3, 4, 5, 6, 7, 8, or 9.

39. $316 \div 289$
40. $662 \div 289$
41. $728 \div 355$
42. $728 \div 177$
43. $921 \div 243$
44. $921 \div 442$
45. $673 \div 109$
46. $673 \div 218$

Divide. You shouldn't have any wrong answers because you can always check your results with multiplication.

47. $1{,}440 \div 32$ **48.** $1{,}206 \div 67$ **49.** $\frac{2{,}401}{49}$ **50.** $\frac{4{,}606}{49}$

51. $28\overline{)12{,}096}$ **52.** $28\overline{)96{,}012}$ **53.** $63\overline{)90{,}594}$ **54.** $45\overline{)17{,}595}$

55. $87\overline{)61{,}335}$ **56.** $79\overline{)48{,}032}$ **57.** $45\overline{)135{,}900}$ **58.** $56\overline{)227{,}920}$

Complete the following tables.

59.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
100	25	
100	26	
100	27	
100	28	

60.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
100	25	
101	25	
102	25	
103	25	

Divide. The following division problems all have remainders.

61. $6\overline{)370}$ **62.** $8\overline{)390}$ **63.** $3\overline{)271}$ **64.** $3\overline{)172}$

65. $26\overline{)345}$ **66.** $26\overline{)543}$ **67.** $71\overline{)16{,}620}$ **68.** $71\overline{)33{,}240}$

69. $23\overline{)9{,}250}$ **70.** $23\overline{)20{,}800}$ **71.** $169\overline{)5{,}950}$ **72.** $391\overline{)34{,}450}$

Paying Attention to Instructions The following two problems are intended to give you practice reading, and paying attention to, the instructions that accompany the problems you are working.

73. **a.** Find the sum of 15 and 5.
b. Find the difference of 15 and 5.
c. Find the product of 15 and 5.
d. Find the quotient of 15 and 5.

74. **a.** Find the sum of 220 and 44.
b. Find the difference of 220 and 44.
c. Find the product of 220 and 44.
d. Find the quotient of 220 and 44.

Applying the Concepts

The application problems that follow may involve more than merely division. Some may require addition, subtraction, or multiplication, whereas others may use a combination of two or more operations.

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75. Price per Pound If 2 pounds of a certain kind of fruit cost 98¢, how much does 1 pound cost?

76. Cost of a Dress A dress shop orders 45 dresses for a total of $675. If they paid the same amount for each dress, how much was each dress?

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77. Fitness Walking The guidelines for fitness now indicate that a person who walks 10,000 steps daily is physically fit. According to The Walking Site on the Internet, it takes just over 2,000 steps to walk one mile. If that is the case, how many miles do you need to walk in order to take 10,000 steps?

78. Filling Glasses How many 8-ounce glasses can be filled from three 32-ounce bottles of soda?

79. Filling Glasses How many 5-ounce glasses can be filled from a 32-ounce bottle of milk? How many ounces of milk will be left in the bottle when all the glasses are full?

80. Filling Glasses How many 3-ounce glasses can be filled from a 28-ounce bottle of milk? How many ounces of milk will be left in the bottle when all the glasses are filled?

81. Filling Glasses How many 32-ounce bottles of Coke will be needed to fill sixteen 6-ounce glasses?

82. Filling Glasses How many 28-ounce bottles of 7-Up will be needed to fill fourteen 6-ounce glasses?

83. Cost of Wine If a person paid $192 for 16 bottles of wine, how much did each bottle cost?

84. Miles per Gallon A traveling salesman kept track of his mileage for 1 month. He found that he traveled 1,104 miles and used 48 gallons of gas. How many miles did he travel on each gallon of gas?

85. Milligrams of Calcium Suppose one egg contains 25 milligrams of calcium, a piece of toast contains 40 milligrams of calcium, and a glass of milk contains 300 milligrams of calcium. How many milligrams of calcium are contained in a breakfast that consists of three eggs, two glasses of milk, and four pieces of toast?

86. Milligrams of Iron Suppose a glass of juice contains 3 milligrams of iron and a piece of toast contains 2 milligrams of iron. If Diane drinks two glasses of juice and has three pieces of toast for breakfast, how much iron is contained in the meal?

87. **Smartphones and Video Streaming** Video streaming on some smartphones can average 5 MB (5 megabytes) per minute. If Taylor used 285 MB on video streaming one day, about how many minutes was he watching video on his phone?

88. **Museum Visitors** In 2012 the Getty Center and Museum in Los Angeles, CA, welcomed over 1,200,000 visitors. If attendance was consistent over the 12 months of the year, about how many people visited the museum each month?

Calculator Problems

Find each of the following quotients using a calculator.

89. $305{,}026 \div 698$

90. $771{,}537 \div 949$

91. 18,436,466 divided by 5,678

92. 2,492,735 divided by 2,345

93. The quotient of 603,955 and 695.

94. The quotient of 875,124 and 876.

95. $4{,}903\overline{)27{,}868{,}652}$

96. $3{,}090\overline{)2{,}308{,}230}$

97. **Gallons per Minute** If a 79,768-gallon tank can be filled in 472 minutes, how many gallons enter the tank each minute?

98. **Weight per Case** A truckload of 632 crates of motorcycle parts weighs 30,968 pounds. How much does each of the crates weigh, if they each weigh the same amount?