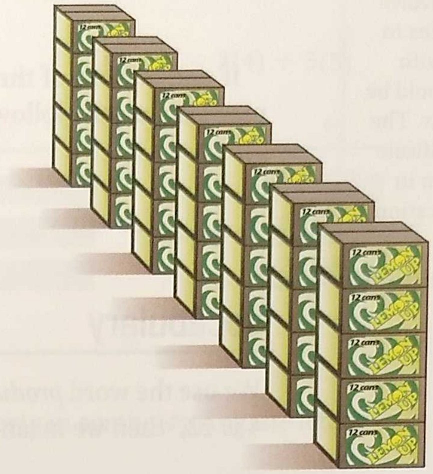
Multiplication with Whole Numbers, and Area

R.5

A supermarket orders 35 cases of a certain soft drink. If each case contains 12 cans of the drink, how many cans were ordered?



To solve this problem and others like it, we must use multiplication. Multiplication is what we will cover in this section.

To begin we can think of multiplication as shorthand for repeated addition. That is, multiplying 3 times 4 can be thought of this way:

$$3 \text{ times } 4 = 4 + 4 + 4 = 12$$

Multiplying 3 times 4 means to add three 4's. We can write 3 times 4 as 3×4, or $3 \cdot 4$.

VIDEO EXAMPLES

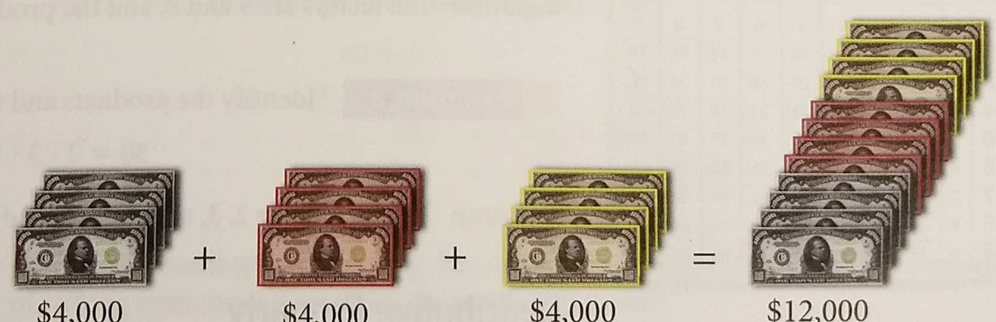
SECTION R.5

Example 1 Multiply $3 \cdot 4{,}000$.

Solution Using the definition of multiplication as repeated addition, we have

$$\begin{aligned} 3 \cdot 4{,}000 &= 4{,}000 + 4{,}000 + 4{,}000 \\ &= 12{,}000 \end{aligned}$$

Here is one way to visualize this process.



Notice that if we had multiplied 3 and 4 to get 12 and then attached three zeros on the right, the result would have been the same.

Notation

There are many ways to indicate multiplication. All the following statements are equivalent. They all indicate multiplication with the numbers 3 and 4.

$$3 \cdot 4, \quad 3 \times 4, \quad 3(4), \quad (3)4, \quad (3)(4), \quad \begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$$

If one or both of the numbers we are multiplying are represented by letters, we may also use the following notation:

$5n$ means 5 times n

ab means a times b

Note The kind of notation we will use to indicate multiplication will depend on the situation. For example, when we are solving equations that involve letters, it is not a good idea to indicate multiplication with the symbol $\times$, since it could be confused with the letter x. The symbol we will use to indicate multiplication most often in this book is the multiplication dot.

Vocabulary

We use the word *product* to indicate multiplication. If we say "The product of 3 and 4 is 12," then we mean

$$3 \cdot 4 = 12$$

Both $3 \cdot 4$ and 12 are called the product of 3 and 4. The 3 and 4 are called *factors*.

In English	In Symbols
The product of 2 and 5	$2 \cdot 5$
The product of 5 and 2	$5 \cdot 2$
The product of 4 and n	$4n$
The product of x and y	xy
The product of 9 and 6 is 54	$9 \cdot 6 = 54$
The product of 2 and 8 is 16	$2 \cdot 8 = 16$

Table 1

Basic Multiplication Facts

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Example 2 Identify the products and factors in the statement

$$9 \cdot 8 = 72$$

Solution The factors are 9 and 8, and the products are $9 \cdot 8$ and 72.

Example 3 Identify the products and factors in the statement

$$30 = 2 \cdot 3 \cdot 5$$

Solution The factors are 2, 3, and 5. The products are $2 \cdot 3 \cdot 5$ and 30.

Distributive Property

To develop an efficient method of multiplication, we need to use what is called the *distributive property*. To begin, consider the following two problems:

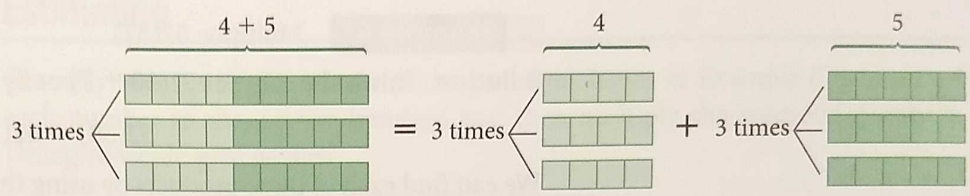
Problem 1	Problem 2
$3(4 + 5)$	$3(4) + 3(5)$
$= 3(9)$	$= 12 + 15$
$= 27$	$= 27$

The result in both cases is the same number, 27. This indicates that the original two expressions must have been equal also. That is,

$$3(4 + 5) = 3(4) + 3(5)$$

This is an example of the distributive property. We say that multiplication *distributes* over addition.

$$3(4 + 5) = 3(4) + 3(5)$$



$$3(4 + 5) \quad = \quad 3 \cdot 4 \quad + \quad 3 \cdot 5$$

We can write this property in symbols using the letters a, b, and c to represent any three whole numbers.

Distributive Property

If a, b, and c represent any three whole numbers, then

$$a(b + c) = a(b) + a(c)$$

Multiplication with Whole Numbers

Suppose we want to find the product 7(65). By writing 65 as 60 + 5 and applying the distributive property, we have:

$$\begin{aligned} 7(65) &= 7(60 + 5) && 65 = 60 + 5 \\ &= 7(60) + 7(5) && \text{Distributive property} \\ &= 420 + 35 && \text{Multiplication} \\ &= 455 && \text{Addition} \end{aligned}$$

We can write the same problem vertically like this:

$$\begin{array}{rrl} & 60 + 5 & \\ \times & 7 & \\ \hline & 35 & \leftarrow 7(5) = 35 \\ + & 420 & \leftarrow 7(60) = 420 \\ \hline & 455 & \end{array}$$

This saves some space in writing. But notice that we can cut down on the amount of writing even more if we write the problem this way:

$$\begin{array}{r} {}^{3} \\ 65 \\ \times\ 7 \\ \hline 455 \end{array}$$

Step 2 7(6) = 42; add the 3 we carried to 42 to get 45

Step 1 7(5) = 35; write the 5 in the ones column, and then carry the 3 to the tens column

This shortcut notation takes some practice.

Example 4 Multiply: 9(43)

Solution

$$\begin{array}{r} ^{2} \\ 43 \\ \times\ 9 \\ \hline 387 \end{array}$$

Step 1 $9(3) = 27$; write the 7 in the ones column, and then carry the 2 to the tens column

Step 2 $9(4) = 36$; add the 2 we carried to 36 to get 38

Example 5 Multiply: 52(37)

Solution This is the same as $52(30 + 7)$ or by the distributive property

$$52(30) + 52(7)$$

We can find each of these products by using the shortcut method:

$$\begin{array}{r} 52 \\ \times\ 30 \\ \hline 1{,}560 \end{array} \qquad \begin{array}{r} ^{1} \\ 52 \\ \times\ 7 \\ \hline 364 \end{array}$$

The sum of these two numbers is $1{,}560 + 364 = 1{,}924$. Here is a summary of what we have so far:

Note This discussion is to show why we multiply the way we do. You should go over it in detail, so you will understand the reasons behind the process of multiplication. Besides being able to do multiplication, you should understand it.

$52(37)$	$= 52(30 + 7)$	$37 = 30 + 7$
	$= 52(30) + 52(7)$	Distributive property
	$= 1{,}560 + 364$	Multiplication
	$= 1{,}924$	Addition

The shortcut form for this problem is

$$\begin{array}{r} 52 \\ \times\ 37 \\ \hline 364 \\ +\ 1{,}560 \\ \hline 1{,}924 \end{array}$$

364 ⟵ $7(52) = 364$

1,560 ⟵ $30(52) = 1{,}560$

In this case we have not shown any of the numbers we carried, simply because it becomes very messy.

Example 6 Multiply: 279(428)

Solution

$$\begin{array}{r} 279 \\ \times\ 428 \\ \hline 2{,}232 \\ 5{,}580 \\ +\ 111{,}600 \\ \hline 119{,}412 \end{array}$$

2,232 ⟵ $8(279) = 2{,}232$

5,580 ⟵ $20(279) = 5{,}580$

111,600 ⟵ $400(279) = 111{,}600$

Using Technology Calculators

Here is how we would work the problem shown in Example 6 on a calculator:

Scientific Calculator 279 [×] 428 [=]

Graphing Calculator 279 [×] 428 [ENT]

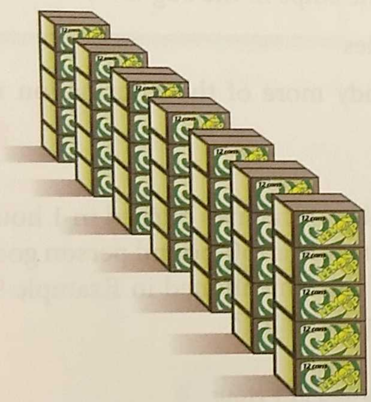
Estimating

One way to estimate the answer to the problem shown in Example 6 is to round each number to the nearest hundred and then multiply the rounded numbers. Doing so would give us this:

$$300(400) = 120{,}000$$

Our estimate of the answer is 120,000, which is close to the actual answer, 119,412. Making estimates is important when we are using calculators; having an estimate of the answer will keep us from making major errors in multiplication.

Applying the Concepts



Example 7 A supermarket orders 35 cases of a certain soft drink. If each case contains 12 cans of the drink, how many cans were ordered?

Solution We have 35 cases and each case has 12 cans. The total number of cans is the product of 35 and 12, which is 35(12):

$$\begin{array}{r} 12 \\ \times\ 35 \\ \hline 60 \\ +\ 360 \\ \hline 420 \end{array} \qquad \begin{array}{l} \\ \\ \longleftarrow 5(12) = 60 \\ \longleftarrow 30(12) = 360 \\ \\ \end{array}$$

There is a total of 420 cans of the soft drink.

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Example 8 Shirley earns $12 an hour for the first 40 hours she works each week. If she has $109 deducted from her weekly check for taxes and retirement, how much money will she take home if she works 38 hours this week?

Solution To find the amount of money she earned for the week, we multiply 12 and 38. From that total we subtract 109. The result is her take-home pay. Without showing all the work involved in the calculations, here is the solution:

$38(\$12) = \$456$ *Her total weekly earnings*

$\$456 - \$109 = \$347$ *Her take-home pay*

Note The letter g that is shown after some of the numbers in the nutrition label in Figure 1 stands for grams, a unit used to measure weight. The unit mg stands for milligrams, another, smaller unit of weight. We will have more to say about these units later in the book.

Example 9 The federal government requires most packaged food to include standardized nutrition information. Figure 1 shows one of these standardized food labels. It is from a package of Fritos Corn Chips that I ate the day I was writing this example. Approximately how many chips are in the bag, and what is the total number of calories consumed if all the chips in the bag are eaten?

Nutrition Facts

Serving Size 1 oz. (28g/About 32 chips)
Servings Per Container: 3

Amount Per Serving	
Calories 160	Calories from fat 90
	% Daily Value*
Total Fat 10 g	16%
Saturated Fat 1.5g	7%
Cholesterol 0mg	0%
Sodium 160mg	7%
Total Carbohydrate 15g	5%
Dietary Fiber 1g	4%
Sugars less than 1g	
Protein 2g	

Figure 1

Solution Reading toward the top of the label, we see that there are about 32 chips in one serving, and approximately 3 servings in the bag. Therefore, the total number of chips in the bag is

$$3(32) = 96 \text{ chips}$$

This is an approximate number, because each serving is approximately 32 chips. Reading further we find that each serving contains 160 calories. Therefore, the total number of calories consumed by eating all the chips in the bag is

$$3(160) = 480 \text{ calories}$$

As we progress through the book, we will study more of the information in nutrition labels.

Example 10 The table below lists the number of calories burned in 1 hour of exercise by a person who weighs 150 pounds. Suppose a 150-pound person goes bowling for 2 hours after having eaten the bag of chips mentioned in Example 9. Will he or she burn all the calories consumed from the chips?

Activity	Calories Burned in 1 Hour by a 150-Pound Person
Bicycling	374
Bowling	265
Handball	680
Jazzercize	340
Jogging	680
Skiing	544

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Solution Each hour of bowling burns 265 calories. If the person bowls for 2 hours, a total of

$$2(265) = 530 \text{ calories}$$

will have been burned. Because the bag of chips contained only 480 calories, all of them have been burned with 2 hours of bowling.

> *Note* To understand some of the notation we use for area, we need to talk about exponents. The 2 in the expression 3^2 is an exponent. The expression 3^2 is read "3 to the second power," or "3 squared," and it is defined this way:
>
> $$3^2 = 3 \cdot 3 = 9$$
>
> As you can see, the exponent 2 in the expression 3^2 tells us to multiply two 3s together. Here are some additional expressions containing the exponent 2.
>
> $$4^2 = 4 \cdot 4 = 16$$
> $$5^2 = 5 \cdot 5 = 25$$
> $$11^2 = 11 \cdot 11 = 121$$
>
> We will cover exponents in more detail later in this chapter.

Area

The *area* of a flat object is a measure of the amount of surface the object has. The rectangle in Figure 2 below has an area of 6 square inches, because that is the number of squares (each of which is 1 inch long and 1 inch wide) it takes to cover the rectangle.

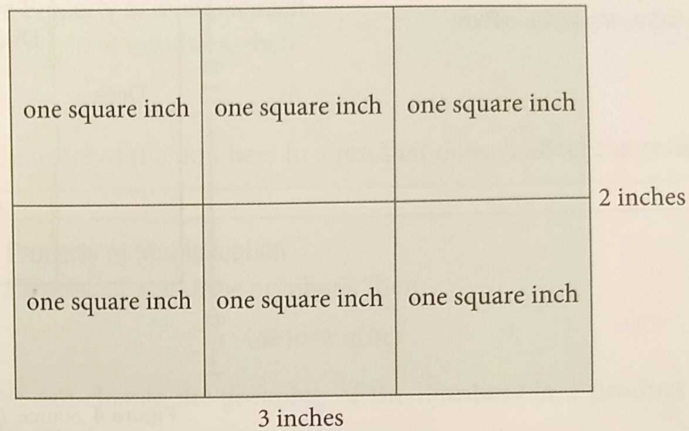


Figure 2 A rectangle with an area of 6 square inches

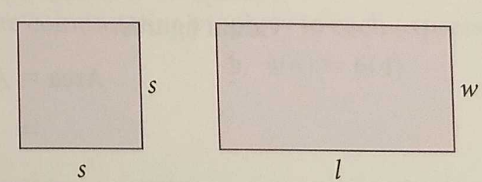
It is no coincidence that the area of the rectangle in Figure 2 and the product of the length and the width are the same number. We can calculate the area of the rectangle in Figure 2 by simply multiplying the length and the width together:

$$\begin{aligned} \text{Area} &= (\text{length}) \cdot (\text{width}) \\ &= (3 \text{ inches}) \cdot (2 \text{ inches}) \\ &= (3 \cdot 2) \cdot (\text{inches} \cdot \text{inches}) \\ &= 6 \text{ square inches} \end{aligned}$$

The unit *square inches* can be abbreviated as *sq. in.* or *in*^{2}.

Facts from Geometry Area

Figure 3 shows two common geometric figures along with the formulas for their areas.



$$\text{Area} = (\text{side})(\text{side}) = (\text{side})^2 = s^2$$

Square

$$\text{Area} = (\text{length})(\text{width}) = lw$$

Rectangle

Figure 3

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Example 11 Find the total area of the house and deck shown in Figure 4.

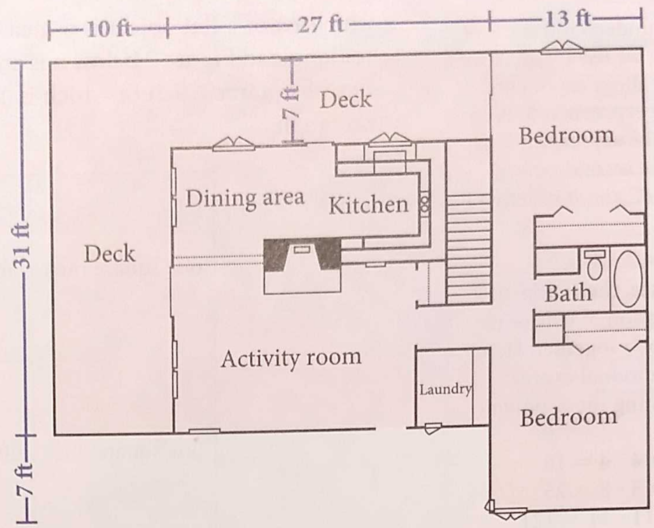


Figure 4 Source: *Image courtesy of COOLhouseplans.com*

Solution We begin by drawing an additional line (shown as a broken line in Figure 5) so that the original figure is now composed of two rectangles. Next, we fill in the missing dimensions on the two rectangles (Figure 6).

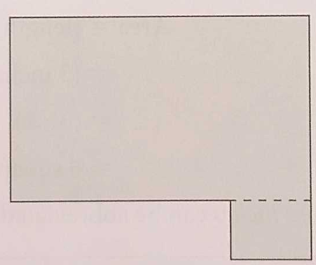


Figure 5

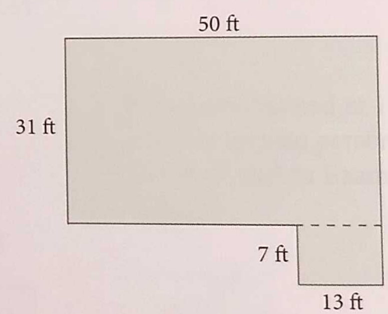


Figure 6

Finally, we calculate the area of the original figure by adding the areas of the individual figures:

$$
\begin{aligned}
\text{Area} &= \text{Area small rectangle} + \text{Area large rectangle} \\
&= 13 \cdot 7 + 50 \cdot 31 \\
&= 91 + 1{,}550 \\
&= 1{,}641 \text{ square feet}
\end{aligned}
$$

More Properties of Multiplication

Multiplication Property of 0

If a represents any number, then

$$a \cdot 0 = 0 \quad \text{and} \quad 0 \cdot a = 0$$

In words Multiplication by 0 always results in 0.

Multiplication Property of 1

If a represents any number, then

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a$$

In words Multiplying any number by 1 leaves that number unchanged.

Commutative Property of Multiplication

If a and b are any two numbers, then

$$ab = ba$$

In words The order of the numbers in a product doesn't affect the result.

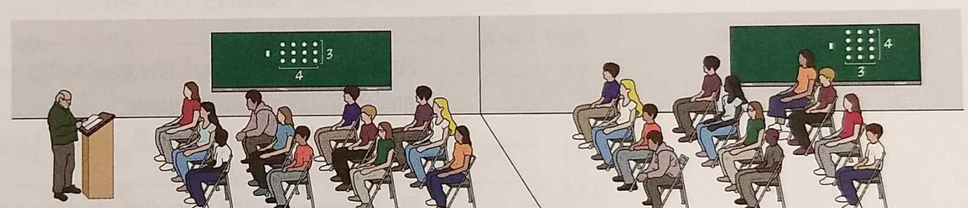
Associative Property of Multiplication

If a, b, and c represent any three numbers, then

$$(ab)c = a(bc)$$

In words We can change the grouping of the numbers in a product without changing the result.

To visualize the commutative property, we can think of an instructor with 12 students. Notice that $3(4) = 4(3) = 12$.



3 chairs across, 4 chairs back = 4 chairs across, 3 chairs back

Example 12 Use the commutative property of multiplication to rewrite each of the following products:

a. $7 \cdot 9$

b. $4(6)$

Solution Applying the commutative property to each expression, we have:

a. $7 \cdot 9 = 9 \cdot 7$

b. $4(6) = 6(4)$

Example 13 Use the associative property of multiplication to rewrite each of the following products:

a. $(2 \cdot 7) \cdot 9$

b. $3 \cdot (8 \cdot 2)$

Solution Applying the associative property of multiplication, we regroup as follows:

a. $(2 \cdot 7) \cdot 9 = 2 \cdot (7 \cdot 9)$

b. $3 \cdot (8 \cdot 2) = (3 \cdot 8) \cdot 2$

Solving Equations

If n is used to represent a number, then the equation

$$4 \cdot n = 12$$

is read "4 times n is 12," or "The product of 4 and n is 12." This means that we are looking for the number we multiply by 4 to get 12. The number is 3. Because the equation becomes a true statement if n is 3, we say that 3 is the solution to the equation.

Example 14 Find the solution to each of the following equations:

a. $6 \cdot n = 24$ **b.** $4 \cdot n = 36$ **c.** $15 = 3 \cdot n$ **d.** $21 = 3 \cdot n$

Solution

a. The solution to $6 \cdot n = 24$ is 4, because $6 \cdot 4 = 24$.

b. The solution to $4 \cdot n = 36$ is 9, because $4 \cdot 9 = 36$.

c. The solution to $15 = 3 \cdot n$ is 5, because $15 = 3 \cdot 5$.

d. The solution to $21 = 3 \cdot n$ is 7, because $21 = 3 \cdot 7$.

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Use the numbers 7, 8, and 9 to give an example of the distributive property.

B. When we write the distributive property in words, we say "multiplication distributes over addition." It is also true that multiplication distributes over subtraction. Use the variables a, b, and c to write the distributive property using multiplication and subtraction.

C. We can multiply 8 and 487 by writing 487 in expanded form as 400 + 80 + 7 and then applying the distributive property. Apply the distributive property to the expression below and then simplify.

$$8(400 + 80 + 7) =$$

D. Find the mistake in the following multiplication problem. Then work the problem correctly.

$$\begin{array}{r} 43 \\ \times\ 68 \\ \hline 344 \\ +\ 258 \\ \hline 602 \end{array}$$

Problem Set R.5

Multiply each of the following.

1. $3 \cdot 100$
2. $7 \cdot 100$
3. $3 \cdot 200$
4. $4 \cdot 200$
5. $6 \cdot 500$
6. $8 \cdot 400$
7. $5 \cdot 1{,}000$
8. $8 \cdot 1{,}000$
9. $3 \cdot 7{,}000$
10. $6 \cdot 7{,}000$
11. $9 \cdot 9{,}000$
12. $7 \cdot 7{,}000$

Find each of the following products. (Multiply.) In each case use the shortcut method.

13. 25×4
14. 43×9
15. 38×6
16. 45×7
17. 18×2
18. 29×3
19. 72×20
20. 68×30
21. 19×50
22. 24×40
23. 69×25
24. 27×36
25. 11×11
26. 12×21
27. 97×16
28. 24×39
29. 168×25
30. 452×34
31. 728×91
32. 680×76
33. 698×400
34. 879×600
35. 111×111
36. 123×321
37. 532×200
38. 277×900
39. 856×232
40. 455×248
41. 976×628
42. 432×555
43. $2{,}468 \times 135$
44. $2{,}725 \times 324$
45. $24{,}563 \times 735$
46. $56{,}728 \times 852$
47. $44{,}777 \times 5{,}888$
48. $33{,}999 \times 2{,}555$

Complete the following tables.

49.

First Number a	Second Number b	Their Product ab
11	11	
11	22	
22	22	
22	44	

50.

First Number a	Second Number b	Their Product ab
25	15	
25	30	
50	15	
50	30	

51.

First Number a	Second Number b	Their Product ab
25	10	
25	100	
25	1,000	
25	10,000	

52.

First Number a	Second Number b	Their Product ab
11	111	
11	222	
22	111	
22	222	

53.

First Number a	Second Number b	Their Product ab
12	20	
36	20	
12	40	
36	40	

54.

First Number a	Second Number b	Their Product ab
10	12	
100	12	
1,000	12	
10,000	12	

Write each of the following expressions in words, using the word *product*.

55. $6 \cdot 7$

56. $9(4)$

57. $2 \cdot n$

58. $5 \cdot x$

59. $9 \cdot 7 = 63$

60. $(5)(6) = 30$

Write each of the following in symbols.

61. The product of 7 and n.

62. The product of 9 and x.

63. The product of 6 and 7 is 42.

64. The product of 8 and 9 is 72.

65. The product of 0 and 6 is 0.

66. The product of 1 and 6 is 6.

Identify the products in each statement.

67. $9 \cdot 7 = 63$

68. $2(6) = 12$

69. $4(4) = 16$

70. $5 \cdot 5 = 25$

Identify the factors in each statement.

71. $2 \cdot 3 \cdot 4 = 24$

72. $6 \cdot 1 \cdot 5 = 30$

73. $12 = 2 \cdot 2 \cdot 3$

74. $42 = 2 \cdot 3 \cdot 7$

Rewrite each of the following using the commutative property of multiplication.

75. $5(9)$

76. $4(3)$

77. $6 \cdot 7$

78. $8 \cdot 3$

Rewrite each of the following using the associative property of multiplication.

79. $2 \cdot (7 \cdot 6)$

80. $4 \cdot (8 \cdot 5)$

81. $3 \times (9 \times 1)$

82. $5 \times (8 \times 2)$

Use the distributive property to rewrite each expression, then simplify.

83. $7(2 + 3)$

84. $4(5 + 8)$

85. $9(4 + 7)$

86. $6(9 + 5)$

87. $3(x + 1)$

88. $5(x + 8)$

89. $2(x + 5)$

90. $4(x + 3)$

Find a solution for each equation.

91. $4 \cdot n = 12$

92. $3 \cdot n = 12$

93. $9 \cdot n = 81$

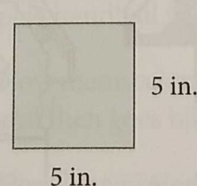
94. $6 \cdot n = 36$

95. $0 = n \cdot 5$

96. $6 = 1 \cdot n$

Find the area enclosed by each figure. (Note that some of the units on the figures come from the metric system. The abbreviations are as follows: Meter is abbreviated m, centimeter is cm, and millimeter is abbreviated mm. A meter is about 3 inches longer than a yard.)

97.



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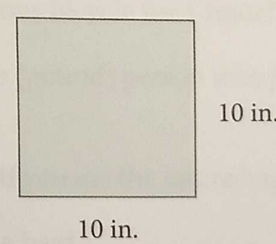

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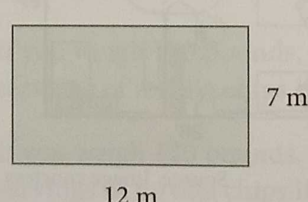

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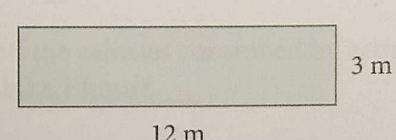

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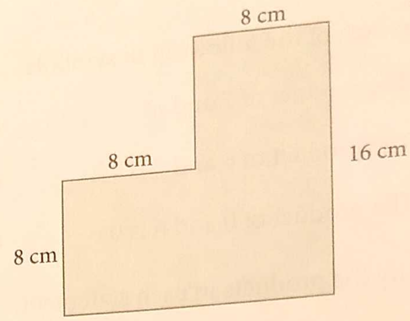

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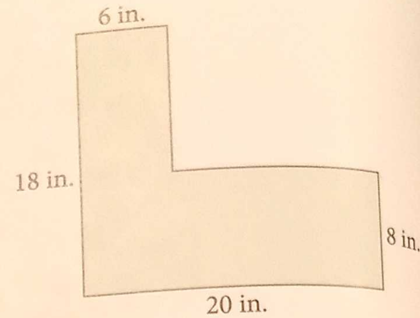

Applying the Concepts

103. Planning a Trip A family decides to drive their compact car on their vacation. They figure it will require a total of about 130 gallons of gas for the vacation. If each gallon of gas will take them 28 miles, how long is the trip they are planning?

104. Rent A student pays $475 rent each month. How much money does she spend on rent in 2 years?


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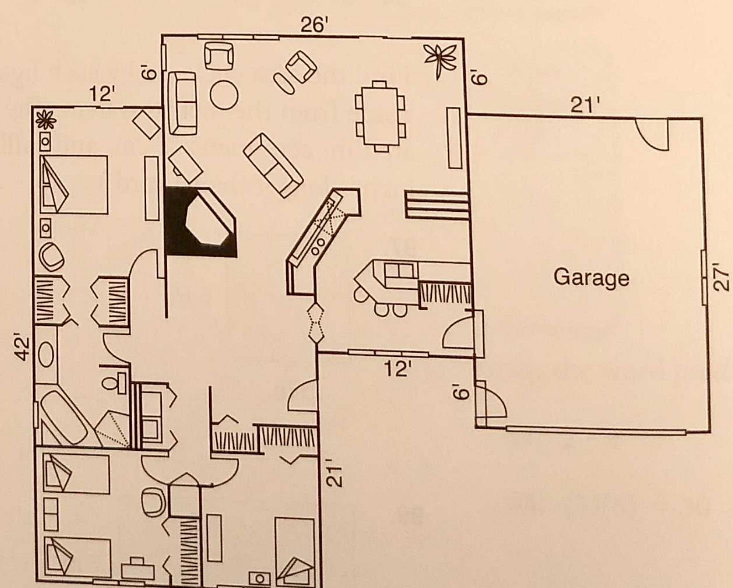
105. Reading House Plans Find the area of the floor of the house shown here if the garage is not included with the house and if the garage is included with the house. (The symbol ' represents feet.)



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