Addition with Whole Numbers, and Perimeter

R.2

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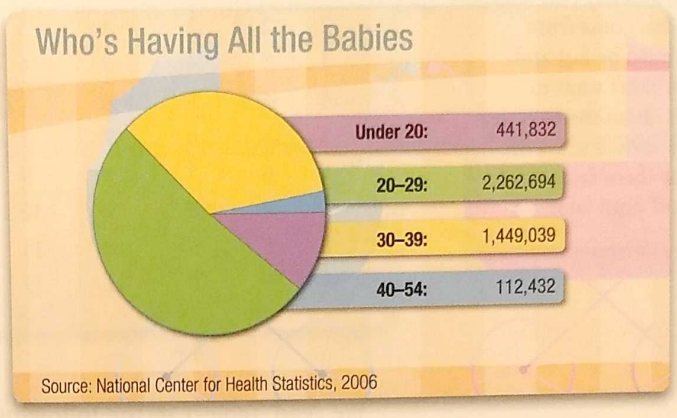
The chart shows the number of babies born in 2006, grouped together according to the age of mothers.



There is much more information available from the table than just the numbers shown. For instance, the chart tells us how many babies were born to mothers less than 30 years of age. But to find that number, we need to be able to do addition with whole numbers. Let's begin by visualizing addition on the number line.

Facts of Addition

Using lengths to visualize addition can be very helpful. In mathematics we generally do so by using the number line. For example, we add 3 and 5 on the number line like this: Start at 0 and move to 3, as shown in Figure 1. From 3, move 5 more units to the right. This brings us to 8. Therefore, $3 + 5 = 8$.

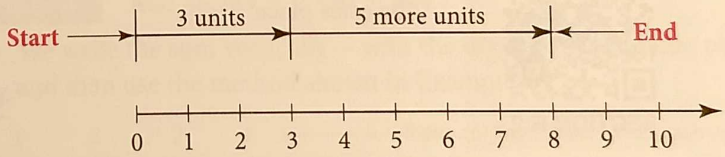


Figure 1

If we do this kind of addition on the number line with all combinations of the numbers 0 through 9, we get the results summarized in Table 1 on the next page.

Addition Table

We call the information in Table 1 our basic addition facts. Your success with the examples and problems in this section depends on knowing the basic addition facts.

Note Table 1 is a summary of the addition facts that you must know in order to make a successful start in your study of basic mathematics. You must know how to add any pair of numbers that come from the list. You must be fast and accurate. You don't want to have to think about the answer to 7 + 9. You should know it's 16. Memorize these facts now. Don't put it off until later.

Addition Table

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Table 1

We read Table 1 in the following manner: Suppose we want to use the table to find the answer to 3 + 5. We locate the 3 in the column on the left and the 5 in the row at the top. We read *across* from the 3 and *down* from the 5. The entry in the table that is across from 3 and below 5 is 8.

Adding Whole Numbers

To add whole numbers, we add digits within the same place value. First we add the digits in the ones place, then the tens place, then the hundreds place, and so on.

VIDEO EXAMPLES

SECTION R.2

Example 1 Add 43 + 52.

Solution This type of addition is best done vertically. First we add the digits in the ones place.

$$\begin{array}{r} 43 \\ +\ 52 \\ \hline 5 \end{array}$$

Then we add the digits in the tens place.

$$\begin{array}{r} 43 \\ +\ 52 \\ \hline 95 \end{array}$$

Note To show why we add digits with the same place value, we can write each number showing the place value of the digits:

$$\begin{array}{r} 43 = 4 \text{ tens} + 3 \text{ ones} \\ +\ 52 = 5 \text{ tens} + 2 \text{ ones} \\ \hline 9 \text{ tens} + 5 \text{ ones} \end{array}$$

Example 2 Add 165 + 801.

Solution Writing the sum vertically, we have

$$\begin{array}{r} 165 \\ +\ 801 \\ \hline 966 \end{array}$$

Add ones place

Add tens place

Add hundreds place

Addition with Carrying

In Examples 1 and 2, the sums of the digits with the same place value were always 9 or less. There are many times when the sum of the digits with the same place value will be a number larger than 9. In these cases we have to do what is called *carrying* in addition. The following examples illustrate this process.

Example 3 Add 197 + 213 + 324.

Solution We write the sum vertically and add digits with the same place value.

```
  1
  197      When we add the ones, we get 7 + 3 + 4 = 14
  213      We write the 4 and carry the 1 to the tens column
+ 324
  4

  11
  197      We add the tens, including the 1 that was carried
  213      over from the last step. We get 13, so we write
+ 324      the 3 and carry the 1 to the hundreds column
  34

  11
  197      We add the hundreds, including the 1 that was
  213      carried over from the last step
+ 324
  734
```

Example 4 Add 46,789 + 2,490 + 864.

Solution We write the sum vertically — with the digits with the same place value aligned — and then use the method shown in Example 3.

```
 1    2     2    1   <-- These are the numbers that have been carried
 4    6  ,  7    8    9
      2  ,  4    9    0
           8    6    4
-------------------------
 5    0  ,  1    4    3
 ^    ^     ^    ^    ^
 |    |     |    |    Write the 3; carry the 1      Ones
 |    |     |    Write the 4; carry the 2           Tens
 |    |     Write the 1; carry the 2                Hundreds
 |    Write the 0; carry the 1                      Thousands
 No carrying necessary                              Ten thousands
```

Adding numbers as we are doing here takes some practice. Most people don't make mistakes in carrying. Most mistakes in addition are made in adding the numbers in the columns. That is why it is so important that you are accurate with the basic addition facts given in this chapter.

Vocabulary

The word we use to indicate addition is the word *sum*. If we say "the sum of 3 and 5 is 8," what we mean is $3 + 5 = 8$. The word *sum* always indicates addition. We can state this fact in symbols by using the letters a and b to represent numbers.

Sum
If a and b are any two numbers, then the **sum** of a and b is $a + b$. To find the sum of two numbers, we add them.

Note When mathematics is used to solve everyday problems, the problems are almost always stated in words. The translation of English to symbols is a very important part of mathematics.

Table 2 gives some phrases and sentences in English and their mathematical equivalents written in symbols.

IN ENGLISH	IN SYMBOLS
The sum of 4 and 1	$4 + 1$
4 added to 1	$1 + 4$
8 more than m	$m + 8$
x increased by 5	$x + 5$
The sum of x and y	$x + y$
The sum of 2 and 4 is 6	$2 + 4 = 6$

Table 2

Properties of Addition

Once we become familiar with addition, we may notice some facts about addition that are true regardless of the numbers involved. The first of these facts involves the number 0 (zero).

Whenever we add 0 to a number, the result is the original number. For example,

$$7 + 0 = 7 \quad \text{and} \quad 0 + 3 = 3$$

Because this fact is true no matter what number we add to 0, we call it a property of 0.

Note When we use letters to represent numbers, as we do when we say "If a and b are any two numbers," then a and b are called variables, because the values they take on vary. We use the variables a and b in the definitions and properties here because we want you to know that the definitions and properties are true for all numbers that you will encounter in this book.

Addition Property of 0
If we let a represent any number, then it is always true that

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

In words Adding 0 to any number leaves that number unchanged.

A second property we notice by becoming familiar with addition is that the order of two numbers in a sum can be changed without changing the result.

$$3 + 5 = 8 \quad \text{and} \quad 5 + 3 = 8$$

$$4 + 9 = 13 \quad \text{and} \quad 9 + 4 = 13$$

This fact about addition is true for *all* numbers. The order in which you add two numbers doesn't affect the result. We call this fact the *commutative property of addition*, and we write it in symbols as follows.

Commutative Property of Addition
If a and b are any two numbers, then it is always true that

$$a + b = b + a$$

In words Changing the order of two numbers in a sum doesn't change the result.

Example 5 Use the commutative property of addition to rewrite each sum.

a. $4 + 6$ **b.** $5 + 9$ **c.** $3 + 0$ **d.** $7 + n$

Solution The commutative property of addition indicates that we can change the order of the numbers in a sum without changing the result. Applying this property we have

a. $4 + 6 = 6 + 4$

b. $5 + 9 = 9 + 5$

c. $3 + 0 = 0 + 3$

d. $7 + n = n + 7$

Notice that we did not actually add any of the numbers. The instructions were to use the commutative property, and the commutative property involves only the order of the numbers in a sum.

Note This discussion is here to show why we write the next property the way we do. Sometimes it is helpful to look ahead to the property itself (in this case, the associative property of addition) to see what it is that is being justified.

The last property of addition we will consider here has to do with sums of more than two numbers. Suppose we want to find the sum of 2, 3, and 4. We could add 2 and 3 first, and then add 4 to what we get

$$(2 + 3) + 4 = 5 + 4 = 9$$

Or, we could add the 3 and 4 together first and then add the 2

$$2 + (3 + 4) = 2 + 7 = 9$$

The result in both cases is the same. If we try this with any other numbers, the same thing happens. We call this fact about addition the *associative property of addition*, and we write it in symbols as follows.

Associative Property of Addition
If a, b, and c represent any three numbers, then

$$(a + b) + c = a + (b + c)$$

In words Changing the grouping of three or more numbers in a sum doesn't change the result.

Example 6 Use the associative property of addition to rewrite each sum.

a. $(5 + 6) + 7$ **b.** $(3 + 9) + 1$ **c.** $6 + (8 + 2)$ **d.** $4 + (9 + n)$

Solution The associative property of addition indicates that we are free to regroup the numbers in a sum without changing the result.

a. $(5 + 6) + 7 = 5 + (6 + 7)$

b. $(3 + 9) + 1 = 3 + (9 + 1)$

c. $6 + (8 + 2) = (6 + 8) + 2$

d. $4 + (9 + n) = (4 + 9) + n$

The commutative and associative properties of addition tell us that when adding whole numbers, we can use any order and grouping. When adding several numbers, it is sometimes easier to look for pairs of numbers whose sums are 10, 20, and so on.

Example 7 Add $9 + 3 + 2 + 7 + 1$.

Solution

$$9 + 3 + 2 + 7 + 1$$

$$10 + 10 + 2$$

$$22$$

Solving Equations

We can use the addition table to help solve some simple equations. If n is used to represent a number, then the equation

$$n + 3 = 5$$

will be true if n is 2. The number 2 is therefore called a *solution* to the equation, because, when we replace n with 2, the equation becomes a true statement:

$$2 + 3 = 5$$

Note The letter n as we are using it here is a variable, because it represents a number. In this case it is the number that is a solution to an equation.

Equations like this are really just puzzles, or questions. When we say, "Solve the equation $n + 3 = 5$," we are asking the question, "What number do we add to 3 to get 5?"

When we solve equations by reading the equation to ourselves and then stating the solution, as we did with the equation above, we are solving the equation by inspection.

Example 8 Find the solution to each equation by inspection.

a. $n + 5 = 9$ **b.** $n + 6 = 12$ **c.** $4 + n = 5$ **d.** $13 = n + 8$

Solution We find the solution to each equation by using the addition facts given in Table 1.

a. The solution to $n + 5 = 9$ is 4, because $4 + 5 = 9$.

b. The solution to $n + 6 = 12$ is 6, because $6 + 6 = 12$.

c. The solution to $4 + n = 5$ is 1, because $4 + 1 = 5$.

d. The solution to $13 = n + 8$ is 5, because $13 = 5 + 8$.

Facts from Geometry Perimeter

We end this section with an introduction to perimeter. Let's start with the definition of a *polygon:*

Polygon

A **polygon** is a closed geometric figure, with at least three sides, in which each side is a straight line segment.

The most common polygons are squares, rectangles, and triangles. Examples of these are shown in Figure 2.

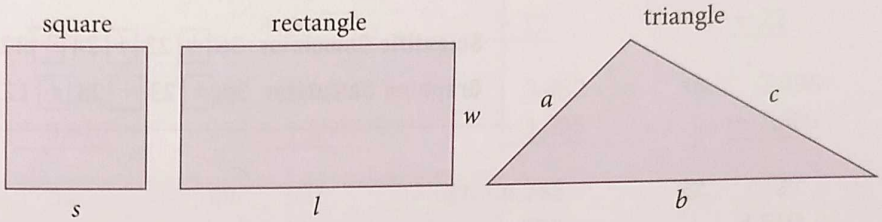


Figure 2

In the square, s is the length of the side, and each side has the same length. In the rectangle, l stands for the length, and w stands for the width. The width is usually the lesser of the two.

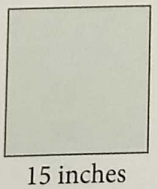
Perimeter

The **perimeter** of any polygon is the sum of the lengths of the sides, and it is denoted with the letter P.

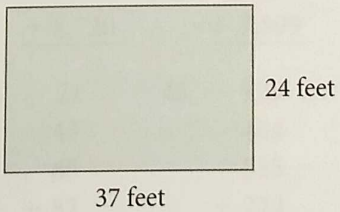
To find the perimeter of a polygon we add all the lengths of the sides together.

Example 9 Find the perimeter of each geometric figure.

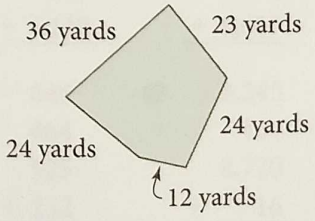
a.



b.



c.



Solution In each case we find the perimeter by adding the lengths of all the sides.

a. The figure is a square. Because the length of each side in the square is the same, the perimeter is

$$P = 15 + 15 + 15 + 15 = 60 \text{ inches}$$

b. In the rectangle, two of the sides are 24 feet long, and the other two are 37 feet long. The perimeter is the sum of the lengths of the sides.

$$P = 24 + 24 + 37 + 37 = 122 \text{ feet}$$

c. For this polygon, we add the lengths of the sides together. The result is the perimeter.

$$P = 36 + 23 + 24 + 12 + 24 = 119 \text{ yards}$$

Using Technology Calculators

From time to time we will include some notes like this one, which show how a calculator can be used to assist us with some of the calculations in the book. Most calculators on the market today fall into one of two categories: those with algebraic logic and those with function logic. Calculators with algebraic logic have a key with an equals sign on it. Calculators with function logic do not have an equals key. Instead they have a key labeled ENTER or EXE (for execute). Scientific calculators use algebraic logic, and graphing calculators, such as the TI-83, use function logic.

Here are the sequences of keystrokes to use to work the problem shown in Part c of Example 9.

Scientific Calculator 36 [+] 23 [+] 24 [+] 12 [+] 24 [=]

Graphing Calculator 36 [+] 23 [+] 24 [+] 12 [+] 24 [ENT]

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. What number is the sum of 6 and 8?

B. Make up an addition problem using the number 456 that does not involve carrying.

C. Make up an addition problem using the number 456 that involves carrying from the ones column to the tens column only.

D. What is the perimeter of a polygon?

Problem Set R.2

Find each of the following sums. (Add.)

1. $3 + 5 + 7$
2. $2 + 8 + 6$
3. $1 + 4 + 9$
4. $2 + 8 + 3$
5. $5 + 9 + 4 + 6$
6. $8 + 1 + 6 + 2$
7. $1 + 2 + 3 + 4 + 5$
8. $5 + 6 + 7 + 8 + 9$
9. $9 + 1 + 8 + 2$
10. $7 + 3 + 6 + 4$

Add each of the following. (There is no carrying involved in these problems.)

33-51 ODD
47-73 ODD
85-91 ODD

11. $\begin{array}{r} 43 \\ +\ 25 \\ \hline \end{array}$
12. $\begin{array}{r} 56 \\ +\ 23 \\ \hline \end{array}$
13. $\begin{array}{r} 81 \\ +\ 17 \\ \hline \end{array}$
14. $\begin{array}{r} 37 \\ +\ 22 \\ \hline \end{array}$
15. $\begin{array}{r} 4{,}281 \\ +\ 3{,}016 \\ \hline \end{array}$
16. $\begin{array}{r} 2{,}749 \\ +\ 1{,}250 \\ \hline \end{array}$
17. $\begin{array}{r} 3{,}482 \\ +\ 3{,}005 \\ \hline \end{array}$
18. $\begin{array}{r} 2{,}496 \\ +\ 7{,}503 \\ \hline \end{array}$
19. $\begin{array}{r} 32 \\ 21 \\ +\ 43 \\ \hline \end{array}$
20. $\begin{array}{r} 521 \\ 340 \\ +\ 135 \\ \hline \end{array}$
21. $\begin{array}{r} 6{,}245 \\ 203 \\ +\ 1{,}001 \\ \hline \end{array}$
22. $\begin{array}{r} 27 \\ 4{,}510 \\ +\ 342 \\ \hline \end{array}$

Add each of the following. (All problems involve carrying in at least one column.)

23. $\begin{array}{r} 49 \\ +\ 16 \\ \hline \end{array}$
24. $\begin{array}{r} 85 \\ +\ 29 \\ \hline \end{array}$
25. $\begin{array}{r} 74 \\ +\ 28 \\ \hline \end{array}$
26. $\begin{array}{r} 36 \\ +\ 46 \\ \hline \end{array}$
27. $\begin{array}{r} 682 \\ +\ 193 \\ \hline \end{array}$
28. $\begin{array}{r} 439 \\ +\ 270 \\ \hline \end{array}$
29. $\begin{array}{r} 638 \\ +\ 191 \\ \hline \end{array}$
30. $\begin{array}{r} 444 \\ +\ 595 \\ \hline \end{array}$
31. $\begin{array}{r} 4{,}963 \\ +\ 5{,}428 \\ \hline \end{array}$
32. $\begin{array}{r} 8{,}291 \\ +\ 7{,}489 \\ \hline \end{array}$
33. $\begin{array}{r} 6{,}205 \\ +\ 9{,}999 \\ \hline \end{array}$
34. $\begin{array}{r} 8{,}888 \\ +\ 9{,}999 \\ \hline \end{array}$
35. $\begin{array}{r} 56{,}789 \\ +\ 98{,}765 \\ \hline \end{array}$
36. $\begin{array}{r} 45{,}678 \\ +\ 87{,}654 \\ \hline \end{array}$
37. $\begin{array}{r} 52{,}468 \\ +\ 58{,}642 \\ \hline \end{array}$
38. $\begin{array}{r} 13{,}579 \\ +\ 97{,}531 \\ \hline \end{array}$
39. $\begin{array}{r} 4{,}296 \\ 8{,}720 \\ +\ 8{,}720 \\ \hline \end{array}$
40. $\begin{array}{r} 5{,}637 \\ 481 \\ +\ 7{,}899 \\ \hline \end{array}$
41. $\begin{array}{r} 4{,}994 \\ 449 \\ +\ 9{,}449 \\ \hline \end{array}$
42. $\begin{array}{r} 6{,}824 \\ 371 \\ +\ 4{,}857 \\ \hline \end{array}$
43. $\begin{array}{r} 12 \\ 34 \\ 56 \\ +\ 78 \\ \hline \end{array}$
44. $\begin{array}{r} 21 \\ 43 \\ 65 \\ +\ 87 \\ \hline \end{array}$
45. $\begin{array}{r} 999 \\ 444 \\ 555 \\ +\ 222 \\ \hline \end{array}$
46. $\begin{array}{r} 646 \\ 464 \\ 525 \\ +\ 252 \\ \hline \end{array}$
47. $\begin{array}{r} 9{,}245 \\ 464 \\ 8{,}720 \\ +\ 16 \\ \hline \end{array}$
48. $\begin{array}{r} 45 \\ 9{,}876 \\ 54 \\ +\ 6{,}789 \\ \hline \end{array}$

Complete the following tables.

49.

First Number a	Second Number b	Their Sum $a + b$
61	38	
63	36	
65	34	
67	32	

50.

First Number a	Second Number b	Their Sum $a + b$
10	45	
20	35	
30	25	
40	15	

51.

First Number a	Second Number b	Their Sum $a + b$
9	16	
36	64	
81	144	
144	256	

52.

First Number a	Second Number b	Their Sum $a + b$
25	75	
24	76	
23	77	
22	78	

Rewrite each of the following using the commutative property of addition.

53. $5 + 9$ **54.** $2 + 1$ **55.** $3 + 8$ **56.** $9 + 2$ **57.** $6 + 4$ **58.** $1 + 7$

Rewrite each of the following using the associative property of addition.

59. $(1 + 2) + 3$ **60.** $(4 + 5) + 9$ **61.** $(2 + 1) + 6$ **62.** $(2 + 3) + 8$

63. $1 + (9 + 1)$ **64.** $2 + (8 + 2)$ **65.** $(4 + n) + 1$ **66.** $(n + 8) + 1$

Find a solution for each equation.

67. $n + 6 = 10$ **68.** $n + 4 = 7$ **69.** $n + 8 = 13$ **70.** $n + 6 = 15$

71. $4 + n = 12$ **72.** $5 + n = 7$ **73.** $17 = n + 9$ **74.** $13 = n + 5$

Write each of the following expressions in words. Use the word *sum* in each case.

75. $4 + 9$ **76.** $9 + 4$ **77.** $8 + 1$

78. $9 + 9$ **79.** $2 + 3 = 5$ **80.** $8 + 2 = 10$

Write each of the following in symbols.

81. a. The sum of 5 and 2 **b.** 3 added to 8

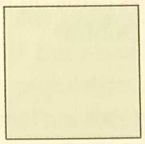
82. a. The sum of a and 4 **b.** 6 more than x

83. a. m increased by 1 **b.** The sum of m and n

84. a. The sum of 4 and 8 is 12. **b.** The sum of a and b is 6.

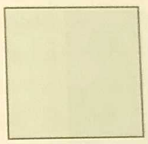
Find the perimeter of each figure. (Note that we have abbreviated the units on each figure to save space. The abbreviation for feet is ft, inches is in., and yards is yd.) The first four figures are squares.

85.



86.

87.



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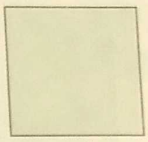

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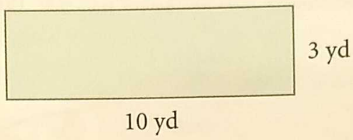

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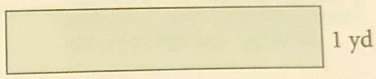

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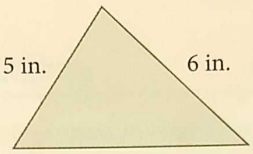

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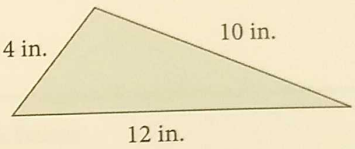
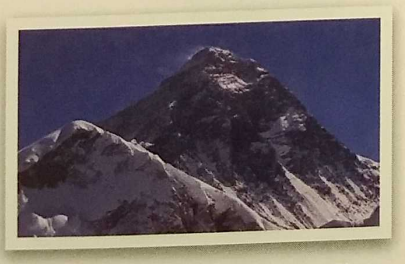


Applying the Concepts

The application problems that follow are related to addition of whole numbers. Read each problem carefully to determine exactly what you are being asked to find. Don't assume that just because a number appears in a problem you have to use it to solve the problem. Sometimes you do, and sometimes you don't.

93. Gallons of Gasoline Tim bought gas for his economy car twice last month. The first time he bought 18 gallons and the second time he bought 16 gallons. What was the total amount of gasoline Tim bought last month?



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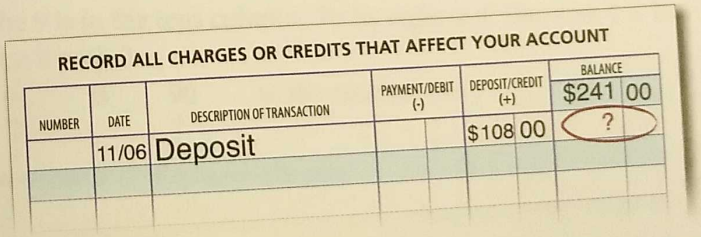
94. Tallest Mountain The world's tallest mountain is Mount Everest. On May 5, 1999, it was found to be 7 feet taller than it was previously thought to be. Before this date, Everest was thought to be 29,028 feet high. That height was determined by B. L. Gulatee in 1954. What is the current height of Mount Everest?

95. Checkbook Balance On Monday Bob had a balance of $241 in his checkbook. On Tuesday he made a deposit of $108. What was the balance in his checkbook on Wednesday?



RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (-)	DEPOSIT/CREDIT (+)	BALANCE
					$241 00
	11/06	Deposit		$108 00	?

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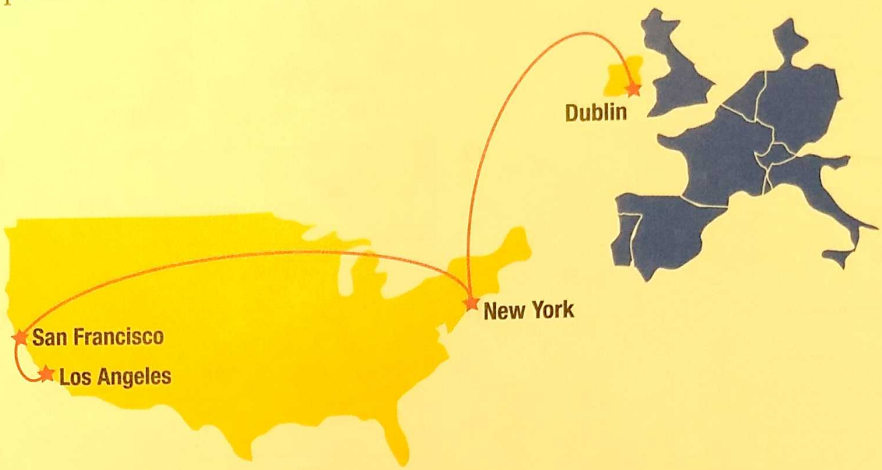
96. Number of Passengers A plane left Los Angeles, with a final destination of Dublin, Ireland, with 67 passengers on board. It made one stop in San Francisco where 28 passengers got on board, and then stopped again in New York, where 57 more passengers came on board. How many passengers were on the plane when it landed in Dublin?



97. Waterpark Slides The Anaconda and Viper are two open flume slides at the Ravine Waterpark in Paso Robles, CA. Each slide is 325 feet long, and the water rushes down at a speed of 1,500 gallons per minute. If you went down the Anaconda and then the Viper, what was the total distance you traveled by inner tube?

98. Water as a Resource According to the Environmental Protection Agency, it takes about 634 gallons of water to produce one hamburger. It takes about 63 gallons of water to produce a glass of milk. If you had a meal of a hamburger and a glass of milk, about how much water was used to produce it?