Multiplication with Fractions, and the Area of a Triangle

2.3

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A recipe calls for $\frac{3}{4}$ cup of flour. If you are making only $\frac{1}{2}$ the recipe, how much flour do you use? This question can be answered by multiplying $\frac{1}{2}$ and $\frac{3}{4}$. Here is the problem written in symbols:

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

As you can see from this example, to multiply two fractions, we multiply the numerators and then multiply the denominators. We begin this section with the rule for multiplication of fractions.

Rule

The product of two fractions is a fraction whose numerator is the product of the two numerators and whose denominator is the product of the two denominators. We can write this rule in symbols as follows:

If a, b, c, and d represent any numbers and b and d are not zero, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

VIDEO EXAMPLES

SECTION 2.3

Example 1 Multiply $\frac{3}{5} \cdot \frac{2}{7}$.

Solution Using our rule for multiplication, we multiply the numerators and multiply the denominators:

$$\frac{3}{5} \cdot \frac{2}{7} = \frac{3 \cdot 2}{5 \cdot 7} = \frac{6}{35}$$

The product of $\frac{3}{4}$ and $\frac{2}{7}$ is the fraction $\frac{6}{35}$. The numerator 6 is the product of 3 and 2, and the denominator 35 is the product of 5 and 7.

Example 2 Multiply $\frac{3}{8} \cdot 5$.

Solution The number 5 can be written as $\frac{5}{1}$. That is, 5 can be considered a fraction with numerator 5 and denominator 1. Writing 5 this way enables us to apply the rule for multiplying fractions.

$$\frac{3}{8} \cdot 5 = \frac{3}{8} \cdot \frac{5}{1}$$

$$= \frac{3 \cdot 5}{8 \cdot 1}$$

$$= \frac{15}{8}$$

Example 3 Multiply $\frac{1}{2}\left(\frac{3}{4} \cdot \frac{1}{5}\right)$.

Solution We find the product inside the parentheses first and then multiply the result by $\frac{1}{2}$:

$$\frac{1}{2}\left(\frac{3}{4} \cdot \frac{1}{5}\right) = \frac{1}{2}\left(\frac{3}{20}\right)$$

$$= \frac{1 \cdot 3}{2 \cdot 20} = \frac{3}{40}$$

The properties of multiplication that we developed in Chapter 1 for whole numbers apply to fractions as well. That is, if a, b, and c are fractions, then

$$a \cdot b = b \cdot a \qquad \text{Multiplication with fractions is commutative}$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \qquad \text{Multiplication with fractions is associative}$$

To demonstrate the associative property for fractions, let's do Example 3 again, but this time we will apply the associative property first:

$$\frac{1}{2}\left(\frac{3}{4} \cdot \frac{1}{5}\right) = \left(\frac{1}{2} \cdot \frac{3}{4}\right) \cdot \frac{1}{5} \qquad \text{Associative property}$$

$$= \left(\frac{1 \cdot 3}{2 \cdot 4}\right) \cdot \frac{1}{5}$$

$$= \left(\frac{3}{8}\right) \cdot \frac{1}{5}$$

$$= \frac{3 \cdot 1}{8 \cdot 5} = \frac{3}{40}$$

The result is identical to that of Example 3.

The answers to all the examples so far in this section have been in lowest terms. Let's see what happens when we multiply two fractions to get a product that is not in lowest terms.

Example 4 Multiply $\frac{15}{8} \cdot \frac{4}{9}$.

Solution Multiplying the numerators and multiplying the denominators, we have

$$\frac{15}{8} \cdot \frac{4}{9} = \frac{15 \cdot 4}{8 \cdot 9}$$

$$= \frac{60}{72}$$

The product is $\frac{60}{72}$, which can be reduced to lowest terms by factoring 60 and 72 and then dividing out any factors they have in common:

$$\frac{60}{72} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot 3}$$

$$= \frac{5}{6}$$

We can actually save ourselves some time by factoring before we multiply. Here's how it is done:

$$\frac{15}{8} \cdot \frac{4}{9} = \frac{15 \cdot 4}{8 \cdot 9}$$

$$= \frac{(3 \cdot 5) \cdot (2 \cdot 2)}{(2 \cdot 2 \cdot 2) \cdot (3 \cdot 3)}$$

$$= \frac{\not{3} \cdot 5 \cdot \not{2} \cdot \not{2}}{\not{2} \cdot \not{2} \cdot 2 \cdot \not{3} \cdot 3}$$

$$= \frac{5}{6}$$

The result is the same in both cases. Reducing to lowest terms before we actually multiply takes less time. Here are some additional examples.

Note Although $\frac{2}{1}$ is in lowest terms, it is still simpler to write the answer as just 2. We will always do this when the denominator is the number 1.

Example 5

$$-\frac{9}{2} \cdot \frac{8}{18} = -\frac{9 \cdot 8}{2 \cdot 18}$$

The product of a negative number and a positive number is a negative number

$$= -\frac{(3 \cdot 3) \cdot (2 \cdot 2 \cdot 2)}{2 \cdot (2 \cdot 3 \cdot 3)}$$

$$= -\frac{\not{3} \cdot \not{3} \cdot \not{2} \cdot \not{2} \cdot 2}{\not{2} \cdot \not{2} \cdot \not{3} \cdot \not{3}}$$

$$= -\frac{2}{1}$$

$$= -2$$

Example 6

$$\frac{2}{3} \cdot \frac{6}{5} \cdot \frac{5}{8} = \frac{2 \cdot 6 \cdot 5}{3 \cdot 5 \cdot 8}$$

$$= \frac{2 \cdot (2 \cdot 3) \cdot 5}{3 \cdot 5 \cdot (2 \cdot 2 \cdot 2)}$$

$$= \frac{\not{2} \cdot \not{2} \cdot \not{3} \cdot \not{5}}{\not{3} \cdot \not{5} \cdot \not{2} \cdot \not{2} \cdot 2}$$

$$= \frac{1}{2}$$

In the previous chapters we did some work with exponents. We can extend our work with exponents to include fractions, as the following examples indicate.

Example 7

$$\left(-\frac{3}{4}\right)^2 = -\frac{3}{4}\left(-\frac{3}{4}\right)$$

The product of two negative numbers is a positive number

$$= \frac{3 \cdot 3}{4 \cdot 4}$$

$$= \frac{9}{16}$$

Example 8 $\left(\frac{5}{6}\right)^2 \cdot \frac{1}{2} = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{2}$

$$= \frac{5 \cdot 5 \cdot 1}{6 \cdot 6 \cdot 2} = \frac{25}{72}$$

Example 9 Simplify each expression.

a. $\left(\frac{2}{3}\right)\left(-\frac{3}{5}\right)$ **b.** $\left(-\frac{7}{8}\right)\left(-\frac{5}{14}\right)$

Solution

a. $\left(\frac{2}{3}\right)\left(-\frac{3}{5}\right) = -\frac{6}{15} = -\frac{2}{5}$ *The rule for multiplication with negative numbers also holds for fractions*

b. $\left(-\frac{7}{8}\right)\left(-\frac{5}{14}\right) = \frac{35}{112} = \frac{5}{16}$

The word *of* used in connection with fractions indicates multiplication. If we want to find $\frac{1}{2}$ of $\frac{2}{3}$, then what we do is multiply $\frac{1}{2}$ and $\frac{2}{3}$.

Example 10 Find $\frac{1}{2}$ of $\frac{2}{3}$.

Solution Knowing the word *of*, as used here, indicates multiplication, we have

$$\frac{1}{2} \text{ of } \frac{2}{3} = \frac{1}{2} \cdot \frac{2}{3}$$

$$= \frac{1 \cdot 2}{2 \cdot 3} = \frac{1}{3}$$

This seems to make sense. Logically, $\frac{1}{2}$ of $\frac{2}{3}$ should be $\frac{1}{3}$, as Figure 1 shows.

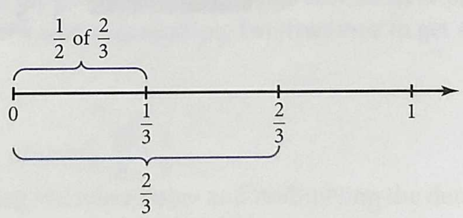

Figure 1

Example 11 What is $\frac{3}{4}$ of 12?

Solution Again, *of* means multiply.

$$\frac{3}{4} \text{ of } 12 = \frac{3}{4}(12)$$

$$= \frac{3}{4}\left(\frac{12}{1}\right)$$

$$= \frac{3 \cdot 12}{4 \cdot 1}$$

$$= \frac{3 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 1} = \frac{9}{1} = 9$$

> **Note on Shortcuts**
>
> As you become familiar with multiplying fractions, you may notice shortcuts that reduce the number of steps in the problems. It's okay to use these shortcuts if you understand why they work and are consistently getting correct answers. If you are using shortcuts and not consistently getting correct answers, then go back to showing all the work until you completely understand the process.

Applying the Concepts

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Example 12 According to a 2009 Pew Research study, about $\frac{2}{5}$ of American teenagers reported that they have been a passenger in a car where the driver used a cell phone in an unsafe manner. In a math class of 35 teenagers, how many would we expect to have had that experience?

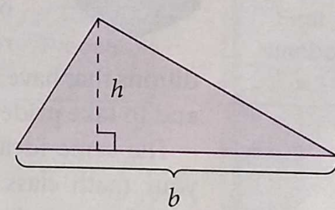
Solution We are looking for the answer to "What is $\frac{2}{5}$ of 35?"

$$\frac{2}{5} \text{ of } 35 = \frac{2}{5}(35)$$
$$= \frac{2}{5}\left(\frac{35}{1}\right)$$
$$= \frac{2 \cdot 35}{5 \cdot 1}$$
$$= \frac{2 \cdot 5 \cdot 7}{5 \cdot 1} = 14$$

In a class of 35 teenagers, we would expect 14 of them to report having been in that situation.

Facts from Geometry The Area of a Triangle

The formula for the area of a triangle is one application of multiplication with fractions. Figure 2 shows a triangle with base b and height h. Below the triangle is the formula for its area. As you can see, it is a product containing the fraction $\frac{1}{2}$.



$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$A = \frac{1}{2}bh$$

Figure 2 The area of a triangle

Example 13 Find the area of the triangle in Figure 3.

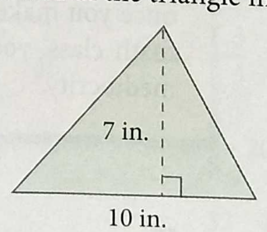


Figure 3 A triangle with base 10 inches and height 7 inches

Solution Applying the formula for the area of a triangle, we have

$$A = \frac{1}{2}bh = \frac{1}{2} \cdot 10 \cdot 7 = 5 \cdot 7 = 35 \text{ in}^2$$

Problem Set 2.3

Find each of the following products. (Multiply.)

1. $\frac{2}{3} \cdot \frac{4}{5}$ **2.** $\frac{5}{6} \cdot \frac{7}{4}$ **3.** $\frac{1}{2} \cdot \frac{7}{4}$ **4.** $\frac{3}{5} \cdot \frac{4}{7}$

5. $-\frac{5}{3}\left(-\frac{3}{5}\right)$ **6.** $-\frac{4}{7}\left(-\frac{7}{4}\right)$ **7.** $\frac{3}{4} \cdot 9$ **8.** $\frac{2}{3} \cdot 5$

9. $\frac{6}{7}\left(\frac{7}{6}\right)$ **10.** $\frac{2}{9}\left(\frac{9}{2}\right)$ **11.** $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$ **12.** $\frac{2}{3} \cdot \frac{4}{5} \cdot \frac{1}{3}$

13. $\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{4}{5}$ **14.** $\frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$ **15.** $\frac{3}{2}\left(-\frac{5}{2}\right)\frac{7}{2}$ **16.** $\frac{4}{3}\left(-\frac{5}{3}\right)\frac{7}{3}$

Complete the following tables.

17.

First Number x	Second Number y	Their Product xy
$\frac{1}{2}$	$\frac{2}{3}$	
$\frac{2}{3}$	$\frac{3}{4}$	
$\frac{3}{4}$	$\frac{4}{5}$	
$\frac{5}{a}$	$\frac{a}{6}$	

18.

First Number x	Second Number y	Their Product xy
12	$\frac{1}{2}$	
12	$\frac{1}{3}$	
12	$\frac{1}{4}$	
12	$\frac{1}{6}$	

19.

First Number x	Second Number y	Their Product xy
$\frac{1}{2}$	30	
$\frac{1}{5}$	30	
$\frac{1}{6}$	30	
$\frac{1}{15}$	30	

20.

First Number x	Second Number y	Their Product xy
$\frac{1}{3}$	$\frac{3}{5}$	
$\frac{3}{5}$	$\frac{5}{7}$	
$\frac{5}{7}$	$\frac{7}{9}$	
$\frac{7}{b}$	$\frac{b}{11}$	

Multiply each of the following. Be sure all answers are written in lowest terms.

21. $\frac{9}{20} \cdot \frac{4}{3}$ **22.** $\frac{135}{16} \cdot \frac{2}{45}$ **23.** $-\frac{3}{4} \cdot 12$ **24.** $-\frac{3}{4} \cdot 20$

25. $-\frac{1}{3}(-3)$ **26.** $-\frac{1}{5}(-5)$ **27.** $\frac{2}{5} \cdot 20$ **28.** $\frac{3}{5} \cdot 15$

29. $\frac{72}{35} \cdot \frac{55}{108} \cdot \frac{7}{110}$ **30.** $\frac{32}{27} \cdot \frac{72}{49} \cdot \frac{1}{40}$

Expand and simplify each of the following.

31. $\left(\frac{2}{3}\right)^2$ **32.** $\left(\frac{3}{5}\right)^2$ **33.** $\left(-\frac{3}{4}\right)^2$ **34.** $\left(-\frac{2}{7}\right)^2$

35. $\left(\frac{1}{2}\right)^2$ **36.** $\left(\frac{1}{3}\right)^2$ **37.** $\left(\frac{2}{3}\right)^3$ **38.** $\left(\frac{3}{5}\right)^3$

39. $\left(\frac{3}{4}\right)^2 \cdot \frac{8}{9}$ **40.** $\left(-\frac{5}{6}\right)^2 \cdot \frac{12}{15}$ **41.** $\left(-\frac{1}{2}\right)^2\left(\frac{3}{5}\right)^2$ **42.** $\left(\frac{3}{8}\right)^2\left(\frac{4}{3}\right)^2$

43. $\left(\frac{1}{2}\right)^2 \cdot 8 + \left(\frac{1}{3}\right)^2 \cdot 9$ **44.** $\left(\frac{2}{3}\right)^2 \cdot 9 + \left(\frac{1}{2}\right)^2 \cdot 4$

45. Find $\frac{3}{8}$ of 64. **46.** Find $\frac{2}{3}$ of 18.

47. What is $\frac{1}{3}$ of the sum of 8 and 4? **48.** What is $\frac{3}{5}$ of the sum of 8 and 7?

49. Find $\frac{1}{2}$ of $\frac{3}{4}$ of 24. **50.** Find $\frac{3}{5}$ of $\frac{1}{3}$ of 15.

Find the mistakes in Problems 51–52. Correct the right-hand side of each one.

51. $\frac{1}{2} \cdot \frac{3}{5} = \frac{4}{10}$ **52.** $\frac{2}{7} \cdot \frac{3}{5} = \frac{5}{35}$

53. **a.** Complete the following table.

b. Using the results of part a, fill in the blank in the following statement:

For numbers larger than 1, the square of the number is ______ than the number.

Number x	Square x^2
1	
2	
3	
4	
5	
6	
7	
8	

54. **a.** Complete the following table.

b. Using the results of part a, fill in the blank in the following statement:

For numbers between 0 and 1, the square of the number is ______ than the number.

Number x	Square x^2
$\frac{1}{2}$	
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{5}$	
$\frac{1}{6}$	
$\frac{1}{7}$	
$\frac{1}{8}$	

Apply the distributive property, then simplify.

55. $4\left(3 + \frac{1}{2}\right)$ **56.** $4\left(2 - \frac{3}{4}\right)$ **57.** $12\left(\frac{1}{2} + \frac{2}{3}\right)$ **58.** $12\left(\frac{3}{4} - \frac{1}{6}\right)$

59. Find the area of the triangle with base 19 inches and height 14 inches.

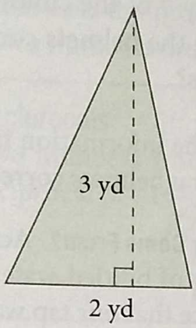
60. Find the area of the triangle with base 13 inches and height 8 inches.

61. The base of a triangle is $\frac{4}{3}$ feet and the height is $\frac{2}{3}$ feet. Find the area.

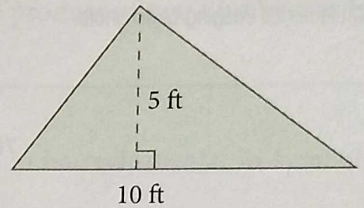
62. The base of a triangle is $\frac{8}{7}$ feet and the height is $\frac{14}{5}$ feet. Find the area.

Find the area of each figure.

63.

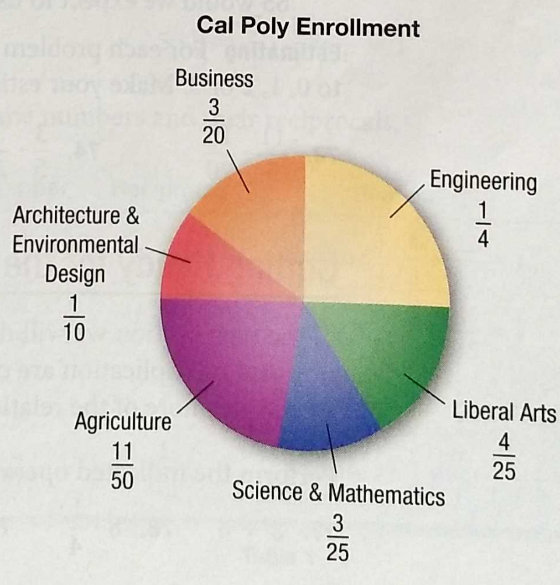


64.



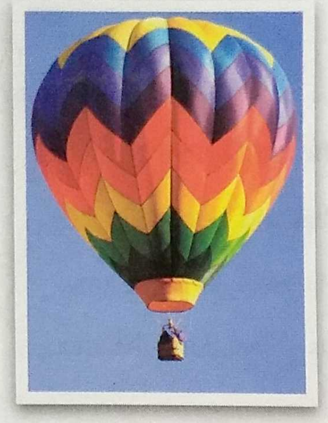
Applying the Concepts

Use the information in the pie chart to answer questions 65 and 66.



65. Reading a Pie Chart If there are approximately 15,800 students attending Cal Poly, approximately how many of them are studying agriculture?

66. Reading a Pie Chart If there are exactly 15,828 students attending Cal Poly, exactly how many of them are studying engineering?



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67. Hot Air Balloon Aerostar International makes a hot air balloon called the Rally 105 that has a volume of 105,400 cubic feet. Another balloon, the Rally 126, was designed with a volume that is approximately $\frac{6}{5}$ the volume of the Rally 105. Find the volume of the Rally 126 to the nearest hundred cubic feet.

68. Health Care According to a report from the Centers for Disease Control for the period 2009-2010, about one-third of the people diagnosed with diabetes don't seek proper medical care. If there are 12 million Americans with diabetes, about how many of them are seeking proper medical care?

69. Bicycle Safety The National Safe Kids Campaign and Bell Sports sponsored a study that surveyed about 8,150 children ages 5 to 14 who were riding bicycles. Approximately $\frac{2}{5}$ of the children were wearing helmets, and of those, only $\frac{13}{20}$ were wearing the helmets correctly. About how many of the children were wearing helmets?

70. Bicycle Safety From the information in Problem 69, how many of the children observed were wearing helmets correctly?

71. Where Does Your Water Come From? According to the Environmental Protection Agency, more than $\frac{1}{4}$ of bottled water comes from a municipal water supply —the very same place that our tap water comes from. In a random collection of 200 water bottles on campus, how many would we expect to contain tap water?

72. Distracted Driving A 2011 Harris Poll found that $\frac{3}{5}$ of drivers admitted to using cell phones while driving. Given this statistic, how many drivers in a group of 65 would we expect to use their cell phones while driving?

Estimating For each problem below, mentally estimate if the answer will be closest to 0, 1, 2 or 3. Make your estimate without using pencil and paper or a calculator.

73. $\frac{11}{5} \cdot \frac{19}{20}$ **74.** $\frac{3}{5} \cdot \frac{1}{20}$ **75.** $\frac{16}{5} \cdot \frac{23}{24}$ **76.** $\frac{9}{8} \cdot \frac{31}{32}$

Getting Ready for the Next Section

In the next section we will do division with fractions. As you already know, division and multiplication are closely related. These review problems are intended to let you see more of the relationship between multiplication and division.

Perform the indicated operations.

77. $8 \div 4$ **78.** $8 \cdot \frac{1}{4}$ **79.** $15 \div 3$ **80.** $15 \cdot \frac{1}{3}$ **81.** $18 \div 6$ **82.** $18 \cdot \frac{1}{6}$

For each number below, find a number to multiply it by to obtain 1.

83. $\frac{3}{4}$ **84.** $\frac{9}{4}$ **85.** $-\frac{1}{3}$ **86.** $\frac{1}{4}$ **87.** 8 **88.** 2