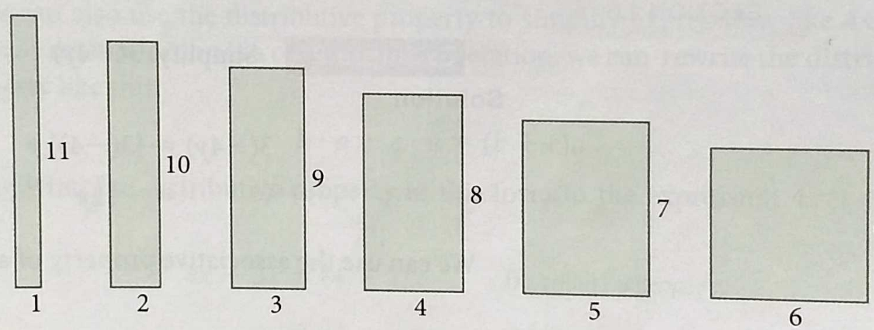
Simplifying Algebraic Expressions

1.6

The woodcut shown here depicts Queen Dido of Carthage in the 9th century B.C., having an ox hide cut into small strips that will be tied together, making a long rope. The rope will be used to enclose her territory. The question, which has become known as the Queen Dido problem, is: what shape will enclose the largest territory?

To translate the problem into something we are more familiar with, suppose we have 24 yards of fencing that we are to use to build a rectangular dog run. If we want the dog run to have the largest area possible then we want the rectangle, with perimeter 24 yards, that encloses the largest area. The diagram below shows six dog runs, each of which has a perimeter of 24 yards. Notice how the length decreases as the width increases.

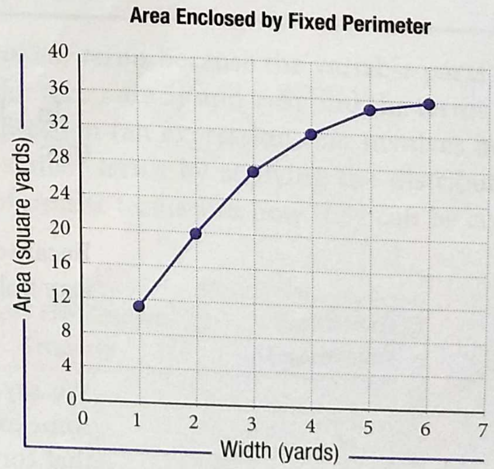
Dog Runs with Perimeter = 24 yards



Since area is length times width, we can build a table and a line graph that show how the area changes as we change the width of the dog run.

Area enclosed by rectangle of perimeter 24 yards

Width (Yards)	Area (Square Yards)
1	11
2	20
3	27
4	32
5	35
6	36



Note At this point, we are interested only in how we can simplify expressions such as $w(12 - w)$. Later, we will see how to use formulas and algebra to come up with those expressions.

We used drawings of possible rectangles to find the related areas. If we were to solve this problem using algebra, we could substitute values for the width in the expression $w(12 - w)$ to obtain the area. In this section we want to simplify expressions containing variables—that is, algebraic expressions such as $w(12 - w)$.

To begin let's review how we use the associative properties for addition and multiplication to simplify expressions.

Consider the expression $4(5x)$. We can apply the associative property of multiplication to this expression to change the grouping so that the 4 and the 5 are grouped together, instead of the 5 and the x. Here's how it looks:

$$4(5x) = (4 \cdot 5)x \quad \text{Associative property}$$
$$= 20x \quad \text{Multiplication: } 4 \cdot 5 = 20$$

We have simplified the expression to $20x$, which in most cases in algebra will be easier to work with than the original expression.

Here are some more examples.

VIDEO EXAMPLES

SECTION 1.6

Example 1 Simplify: $-2(5x)$

Solution

$$-2(5x) = (-2 \cdot 5)x \quad \text{Associative property}$$
$$= -10x \quad \text{The product of } -2 \text{ and } 5 \text{ is } -10$$

Example 2 Simplify: $3(-4y)$

Solution

$$3(-4y) = [3(-4)]\,y \quad \text{Associative property}$$
$$= -12y \quad 3 \text{ times } -4 \text{ is } -12$$

We can use the associative property of addition to simplify expressions also.

Example 3 Simplify: $(2x + 5) + 10$

Solution

$$(2x + 5) + 10 = 2x + (5 + 10) \quad \text{Associative property}$$
$$= 2x + 15 \quad \text{Addition}$$

In Chapter R we introduced the distributive property. In symbols it looks like this:

$$a(b + c) = ab + ac$$

Because subtraction is defined as addition of the opposite, the distributive property holds for subtraction as well as addition. That is,

$$a(b - c) = ab - ac$$

We say that multiplication distributes over addition and subtraction. Here are some examples that review how the distributive property is applied to expressions that contain variables.

Example 4 Simplify: $2(a - 3)$

Solution

$$2(a - 3) = 2(a) - 2(3) \quad \text{Distributive property}$$
$$= 2a - 6 \quad \text{Multiplication}$$

In Examples 1 and 2 we simplified expressions such as $4(5x)$ by using the associative property. Here are some examples that use a combination of the associative property and the distributive property.

Example 5 Simplify: $4(5x + 3)$

Solution

$$4(5x + 3) = 4(5x) + 4(3) \quad \text{Distributive property}$$
$$= (4 \cdot 5)x + 4(3) \quad \text{Associative property}$$
$$= 20x + 12 \quad \text{Multiplication}$$

Example 6 Simplify: $5(2x + 3y)$

Solution

$$5(2x + 3y) = 5(2x) + 5(3y) \quad \text{Distributive property}$$
$$= 10x + 15y \quad \text{Associative property and multiplication}$$

We can also use the distributive property to simplify expressions like $4x + 3x$. Because multiplication is a commutative operation, we can rewrite the distributive property like this:

$$b \cdot a + c \cdot a = (b + c)a$$

Applying the distributive property in this form to the expression $4x + 3x$, we have

$$4x + 3x = (4 + 3)x \quad \text{Distributive property}$$
$$= 7x \quad \text{Addition}$$

Similar Terms

Expressions like $4x$ and $3x$ are called *similar terms* because the variable parts are the same. Some other examples of similar terms are $5y$ and $-6y$ and the terms $7a$, $-13a$, $\frac{3}{4}a$. To simplify an algebraic expression (an expression that involves both numbers and variables), we combine similar terms by applying the distributive property. Table 1 shows several pairs of similar terms and how they can be combined using the distributive property.

Original Expression		Apply Distributive Property		Simplified Expression
$4x + 3x$	=	$(4 + 3)x$	=	$7x$
$7a + a$	=	$(7 + 1)a$	=	$8a$
$-5x + 7x$	=	$(-5 + 7)x$	=	$2x$
$8y - y$	=	$(8 - 1)y$	=	$7y$
$-4a - 2a$	=	$(-4 - 2)a$	=	$-6a$
$3x - 7x$	=	$(3 - 7)x$	=	$-4x$

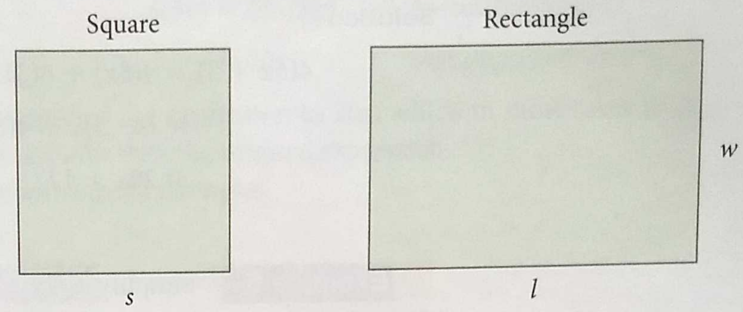
Table 1

Note Combining similar terms is an important part of solving equations in Chapter 3.

As you can see from the table, the distributive property can be applied to any combination of positive and negative terms so long as they are similar terms.

Algebraic Expressions Representing Area and Perimeter

Below are a square with a side of length s and a rectangle with a length of l and a width of w. The table that follows the figures reviews the formulas for the area and perimeter that we introduced in Chapter R.



	Square	Rectangle
Area A	s^2	lw
Perimeter P	$4s$	$2l + 2w$

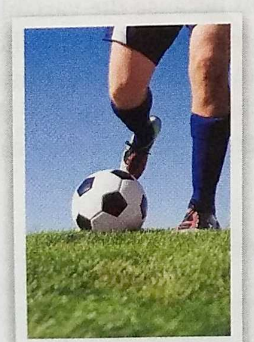
Example 7 Find the area and perimeter of a square with a side 6 inches long.

Solution Substituting 6 for s in the formulas for area and perimeter of a square, we have

$$\text{Area} = A = s^2 = 6^2 = 36 \text{ square inches}$$

$$\text{Perimeter} = P = 4s = 4(6) = 24 \text{ inches}$$

Applying the Concepts



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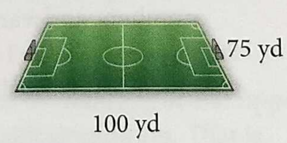
Example 8 A soccer field is 100 yards long and 75 yards wide. Find the area and perimeter.



Solution Substituting 100 for l and 75 for w in the formulas for area and perimeter of a rectangle, we have

$$\text{Area} = A = lw = 100(75) = 7{,}500 \text{ square yards}$$

$$\text{Perimeter} = P = 2l + 2w = 2(100) + 2(75) = 200 + 150 = 350 \text{ yards}$$

Example 9 We can write the expression for the area in Example 8 using algebra as $w(175 - w)$. Simplify this expression and then substitute 75 for the width in the new expression. Do you get the same area as in Example 8?

Solution Using the distributive property, we have

$$w(175 - w) = 175w - w^2$$

Substituting 75 for w, we have

$$175(75) - (75)^2 = 13{,}125 - 5{,}625 = 7{,}500$$

The area in both cases is 7,500 square yards.

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Without actually multiplying, how do you apply the associative property to the expression $4(5x)$?

B. What are similar terms?

C. Explain why $2a - a$ is a, rather than 1.

D. Can two rectangles with the same perimeter have different areas?

Problem Set 1.6

Apply the associative property to each expression, and then simplify the result.

1. $5(4a)$
2. $8(9a)$
3. $6(8a)$
4. $3(2a)$
5. $-6(3x)$
6. $-2(7x)$
7. $-3(9x)$
8. $-4(6x)$
9. $5(-2y)$
10. $3(-8y)$
11. $6(-10y)$
12. $5(-5y)$
13. $2 + (3 + x)$
14. $9 + (6 + x)$
15. $5 + (8 + x)$
16. $3 + (9 + x)$
17. $4 + (6 + y)$
18. $2 + (8 + y)$
19. $7 + (1 + y)$
20. $4 + (1 + y)$
21. $(5x + 2) + 4$
22. $(8x + 3) + 10$
23. $(6y + 4) + 3$
24. $(3y + 7) + 8$
25. $(12a + 2) + 19$
26. $(6a + 3) + 14$
27. $(7x + 8) + 20$
28. $(14x + 3) + 15$

Apply the distributive property to each expression, and then simplify.

29. $7(x + 5)$
30. $8(x + 3)$
31. $6(a - 7)$
32. $4(a - 9)$
33. $2(x - y)$
34. $5(x - a)$
35. $4(5 + x)$
36. $8(3 + x)$
37. $3(2x + 5)$
38. $8(5x + 4)$
39. $6(3a + 1)$
40. $4(8a + 3)$
41. $2(6x - 3y)$
42. $7(5x - y)$
43. $5(7 - 4y)$
44. $8(6 - 3y)$

Use the distributive property to combine similar terms.

45. $3x + 5x$
46. $7x + 8x$
47. $3a + a$
48. $8a + a$
49. $-2x + 6x$
50. $-3x + 9x$
51. $6y - y$
52. $3y - y$
53. $-8a - 2a$
54. $-7a - 5a$
55. $4x - 9x$
56. $5x - 11x$

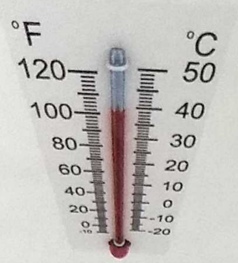
Applying the Concepts

Area and Perimeter Find the area and perimeter of each square if the length of each side is as given below.

57. $s = 6$ feet
58. $s = 14$ yards
59. $s = 9$ inches
60. $s = 15$ meters

Area and Perimeter Find the area and perimeter for a rectangle if the length and width are as given below.

61. $l = 20$ inches, $w = 10$ inches
62. $l = 40$ yards, $w = 20$ yards
63. $l = 25$ feet, $w = 12$ feet
64. $l = 210$ meters, $w = 120$ meters



Temperature Scales In the metric system, the scale we use to measure temperature is the Celsius scale. On this scale water boils at 100 degrees and freezes at 0 degrees. When we write 100 degrees measured on the Celsius scale, we use the notation 100 °C, which is read "100 degrees Celsius." If we know the temperature in degrees Fahrenheit, we can convert to degrees Celsius by using the formula

$$C = \frac{5(F - 32)}{9}$$

where F is the temperature in degrees Fahrenheit. Use this formula to find the temperature in degrees Celsius for each of the following Fahrenheit temperatures.

65. 68 °F **66.** 59 °F **67.** 41 °F **68.** 23 °F **69.** 14 °F **70.** 32 °F