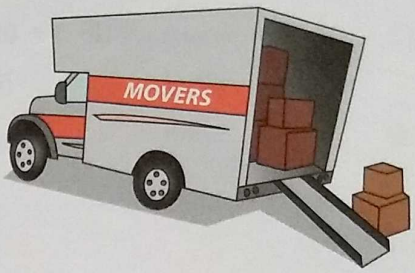
Division with Negative Numbers 1.5



Suppose four friends invest equal amounts of money in a moving truck to start a small business. After 2 years the truck has dropped $10,000 in value. If we represent this change with the number −$10,000, then the loss to each of the four partners can be found with division:

$$(-\$10{,}000) \div 4 = -\$2{,}500$$

From this example it seems reasonable to assume that a negative number divided by a positive number will give a negative answer.

To cover all the possible situations we can encounter with division of negative numbers, we use the relationship between multiplication and division. If we let n be the answer to the problem $12 \div (-2)$, then we know that

$$12 \div (-2) = n \quad \text{and} \quad -2(n) = 12$$

From our work with multiplication, we know that n must be -6 in the multiplication problem above, because -6 is the only number we can multiply -2 by to get 12. Because of the relationship between the two problems above, it must be true that 12 divided by -2 is -6.

The following pairs of problems show more quotients of positive and negative numbers. In each case the multiplication problem on the right justifies the answer to the division problem on the left.

$6 \div 3 = 2$	because	$3(2) = 6$
$6 \div (-3) = -2$	because	$-3(-2) = 6$
$-6 \div 3 = -2$	because	$3(-2) = -6$
$-6 \div (-3) = 2$	because	$-3(2) = -6$

These results can be used to write the rule for division with negative numbers.

Rule

To divide two numbers, we divide their absolute values.

1. The answer is positive if both the original numbers have the same sign. That is, the quotient of two numbers with the same signs is positive.
2. The answer is negative if the original two numbers have different signs. That is, the quotient of two numbers with different signs is negative.

VIDEO EXAMPLES

SECTION 1.5

Example 1 Divide:

a. $12 \div 4$ **b.** $-12 \div 4$ **c.** $12 \div (-4)$ **d.** $-12 \div (-4)$

Solution

a. $12 \div 4 = 3$ Like signs; positive answer

b. $-12 \div 4 = -3$ Unlike signs; negative answer

c. $12 \div (-4) = -3$ Unlike signs; negative answer

d. $-12 \div (-4) = 3$ Like signs; positive answer

Example 2 Simplify:

a. $\frac{20}{5}$ **b.** $\frac{-20}{5}$ **c.** $\frac{20}{-5}$ **d.** $\frac{-20}{-5}$

Solution

a. $\frac{20}{5} = 4$ Like signs; positive answer

b. $\frac{-20}{5} = -4$ Unlike signs; negative answer

c. $\frac{20}{-5} = -4$ Unlike signs; negative answer

d. $\frac{-20}{-5} = 4$ Like signs; positive answer

From the examples we have done so far, we can make the following generalization about quotients that contain negative signs:

If a and b are numbers and b is not equal to 0, then

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \quad \text{and} \quad \frac{-a}{-b} = \frac{a}{b}$$

The last examples in this section involve more than one operation. We use the rules developed previously in this chapter and the rule for order of operations to simplify each.

Example 3 Simplify: $\frac{-15 + 5(-4)}{12 - 17}$

Solution Simplifying above and below the fraction bar, we have

$$\frac{-15 + 5(-4)}{12 - 17} = \frac{-15 + (-20)}{-5} = \frac{-35}{-5} = 7$$

Example 4 Simplify: $-4(10^2) + 20 \div (-4)$

Solution Applying the rule for order of operations, we have

$$\begin{aligned} -4(10^2) + 20 \div (-4) &= -4(100) + 20 \div (-4) && \text{Exponents first} \\ &= -400 + (-5) && \text{Multiply and divide} \\ &= -405 && \text{Add} \end{aligned}$$

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Write a multiplication problem that is equivalent to the division problem $-12 \div 4 = -3$.

B. Write a multiplication problem that is equivalent to the division problem $-12 \div (-4) = 3$.

C. If two numbers have the same sign, then their quotient will have what sign?

D. Dividing a negative number by 0 always results in what kind of expression?

Problem Set 1.5

Find each of the following quotients. (Divide.)

1. $-15 \div 5$
2. $15 \div (-3)$
3. $20 \div (-4)$
4. $-20 \div 4$
5. $-30 \div (-10)$
6. $-50 \div (-25)$
7. $\dfrac{-14}{-7}$
8. $\dfrac{-18}{-6}$
9. $\dfrac{12}{-3}$
10. $\dfrac{12}{-4}$
11. $-22 \div 11$
12. $-35 \div 7$
13. $\dfrac{0}{-3}$
14. $\dfrac{0}{-5}$
15. $125 \div (-25)$
16. $-144 \div (-9)$

Complete the following tables.

17.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
100	−5	
100	−10	
100	−25	
100	−50	

18.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
24	−4	
24	−3	
24	−2	
24	−1	

19.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
−100	−5	
−100	5	
100	−5	
100	5	

20.

First Number a	Second Number b	The Quotient of a and b $\frac{a}{b}$
−24	−2	
−24	−4	
−24	−6	
−24	−8	

Use any of the rules developed in this chapter and the rule for order of operations to simplify each of the following expressions as much as possible.

21. $\dfrac{4(-7)}{-28}$
22. $\dfrac{6(-3)}{-18}$
23. $\dfrac{-3(-10)}{-5}$
24. $\dfrac{-4(-12)}{-6}$
25. $\dfrac{2(-3)}{6-3}$
26. $\dfrac{2(-3)}{3-6}$
27. $\dfrac{4-8}{8-4}$
28. $\dfrac{9-5}{5-9}$
29. $\dfrac{2(-3)+10}{-4}$
30. $\dfrac{7(-2)-6}{-10}$
31. $\dfrac{2+3(-6)}{4-12}$
32. $\dfrac{3+9(-1)}{5-7}$
33. $\dfrac{6(-7)+3(-2)}{20-4}$
34. $\dfrac{9(-8)+5(-1)}{12-1}$
35. $\dfrac{3(-7)(-4)}{6(-2)}$
36. $\dfrac{-2(4)(-8)}{(-2)(-2)}$

37. $(-5)^2 + 20 \div 4$

38. $6^2 + 36 \div 9$

39. $100 \div (-5)^2$

40. $400 \div (-4)^2$

41. $-100 \div 10 \div 2$

42. $-500 \div 50 \div 10$

43. $-100 \div (10 \div 2)$

44. $-500 \div (50 \div 10)$

45. $(-100 \div 10) \div 2$

46. $(-500 \div 50) \div 10$

Paying Attention to Instructions The following two problems are intended to give you practice reading, and paying attention to, the instructions that accompany the problems you are working.

47. **a.** Find the sum of -220 and 44.
b. Find the difference of -220 and 44.
c. Find the product of -220 and 44.
d. Find the quotient of -220 and 44.

48. **a.** Find the sum of -15 and -5.
b. Find the difference of -15 and -5.
c. Find the product of -15 and -5.
d. Find the quotient of -15 and -5.

49. What number do you divide by -5 to get -7?

50. What number do you divide by 6 to get -7?

51. Subtract -3 from the quotient of 27 and 9.

52. Subtract -7 from the quotient of -72 and -9.

Estimating

Work Problems 53–60 mentally, without pencil and paper or a calculator.

53. Is $397 \div (-401)$ closer to 1 or -1?

54. Is $-751 \div (-749)$ closer to 1 or -1?

55. The quotient $-121 \div 27$ is closest to which of the following numbers?

a. -150 **b.** -100 **c.** -4 **d.** 6

56. The quotient $1{,}000 \div (-337)$ is closest to which of the following numbers?

a. 663 **b.** -3 **c.** -30 **d.** -663

57. Which number is closest to the sum $-151 + (-49)$?

a. -200 **b.** -100 **c.** 3 **d.** 7,500

58. Which number is closest to the difference $-151 - (-49)$?

a. -200 **b.** -100 **c.** 3 **d.** 7,500

59. Which number is closest to the product $-151(-49)$?

a. -200 **b.** -100 **c.** 3 **d.** 7,500

60. Which number is closest to the quotient $-151 \div (-49)$?

a. -200 **b.** -100 **c.** 3 **d.** 7,500

Applying the Concepts

61. Mean Find the mean of the numbers $-5, 0, 5$.

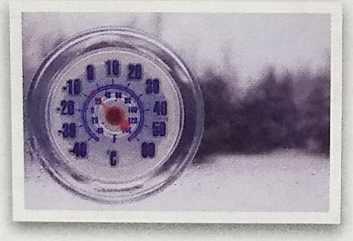
62. Median Find the median of the numbers $-5, 0, 5$.

63. Averages For the numbers $-5, -4, 0, 2, 2, 2, 3$, find

a. The mean **b.** The median **c.** The mode

64. Averages For the numbers $-5, -2, -2, 0, 2, 7$, find

a. The mean **b.** The median **c.** The mode



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65. Temperature Line Graph The table below gives the low temperature for each day of one week in White Bear Lake, Minnesota. Draw a line graph of the information in the table.

Low Temperatures in White Bear Lake, Minnesota

Day	Temperature
Monday	10 °F
Tuesday	8 °F
Wednesday	−5 °F
Thursday	−3 °F
Friday	−8 °F
Saturday	5 °F
Sunday	7 °F

66. Temperature Line Graph The table below gives the low temperature for each day of one week in Fairbanks, Alaska. Draw a line graph of the information in the table.

Low Temperatures in Fairbanks, Alaska

Day	Temperature
Monday	−26 °F
Tuesday	−5 °F
Wednesday	9 °F
Thursday	12 °F
Friday	3 °F
Saturday	−15 °F
Sunday	−20 °F

67. Average Temperatures Use the information in the table in Problem 65 to find

a. the mean low temperature for the week

b. the median low temperature for the week

68. Average Temperatures Use the information in the table in Problem 66 to find

a. the mean low temperature for the week

b. the median low temperature for the week

69. Average Depth A marine research submarine collected samples of seawater at 54 feet below the ocean's surface, 119 feet below, 267 feet below, and 400 feet below. What was the average depth of the samples?

70. Average Stock Gain/Loss A stock had the following dollar gains and losses for a week: $+3$, -5, -3, $+2$, $+2$, -4, and -2. What was the average gain or loss for that week?

Getting Ready for the Next Section

The problems below review some of the properties of addition and multiplication we covered in Chapter R.

Rewrite each expression using the commutative property of addition or multiplication.

71. $3 + x$ **72.** $4y$

Rewrite each expression using the associative property of addition or multiplication.

73. $5 + (7 + a)$ **74.** $(2x + 5) + 10$ **75.** $3(4y)$ **76.** $2(5x)$

Apply the distributive property to each expression.

77. $5(3 + 7)$

78. $8(4 + 2)$

Simplify.

79. 6^2

80. 12^2

81. $100(75)$

82. $100(53)$

Add or subtract according to the rules for positive and negative numbers.

83. $2(100) + 2(75)$

84. $2(100) + 2(53)$

85. **a.** $4 + 3$
b. $-5 + 7$
c. $8 - 1$
d. $-4 - 2$
e. $3 - 7$

86. **a.** $5 + 2$
b. $-6 + 7$
c. $9 - 1$
d. $-5 - 3$
e. $2 - 5$