Multiplication with Negative Numbers

1.4

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Suppose you buy three shares of a stock on Monday, and by Friday the price per share has dropped $5. How much money have you lost? The answer is $15. Because it is a loss, we can express it as −$15. The multiplication problem below can be used to describe the relationship among the numbers.

3 shares each loses $5 for a total of −$15

$$3(-5) = -15$$

From this we conclude that it is reasonable to say that the product of a positive number and a negative number is a negative number.

In order to generalize multiplication recall that we first defined multiplication by whole numbers to be repeated addition. That is:

$$3 \cdot 5 = 5 + 5 + 5$$

Multiplication Repeated addition

This concept is very helpful when it comes to developing the rule for multiplication problems that involve negative numbers. For the first example we look at what happens when we multiply a negative number by a positive number.

VIDEO EXAMPLES

SECTION 1.4

Example 1 Multiply: $3(-5)$

Solution Writing this product as repeated addition, we have

$$\begin{aligned} 3(-5) &= (-5) + (-5) + (-5) \\ &= -10 + (-5) \\ &= -15 \end{aligned}$$

The result, −15, is obtained by adding the three negative 5's.

Example 2 Multiply: $-3(5)$

Solution In order to write this multiplication problem in terms of repeated addition, we will have to reverse the order of the two numbers. This is easily done, because multiplication is a commutative operation.

$$\begin{aligned} -3(5) &= 5(-3) && \text{Commutative property} \\ &= (-3) + (-3) + (-3) + (-3) + (-3) && \text{Repeated addition} \\ &= -15 && \text{Addition} \end{aligned}$$

The product of −3 and 5 is −15.

Example 3 Multiply: $-3(-5)$

Solution It is impossible to write this product in terms of repeated addition. We will find the answer to $-3(-5)$ by solving a different problem. Look at the following problem:

$$-3[5 + (-5)] = -3[0] = 0$$

The result is 0, because multiplying by 0 always produces 0. Now we can work the same problem another way and in the process find the answer to $-3(-5)$. Applying the distributive property to the same expression, we have

$$-3[5 + (-5)] = -3(5) + (-3)(-5) \qquad \text{Distributive property}$$
$$= -15 + (?) \qquad -3(5) = -15$$

Note The discussion here explains why $-3(-5) = 15$. We want to be able to justify everything we do in mathematics.

The question mark must be $+15$, because we already know that the answer to the problem is 0, and $+15$ is the only number we can add to -15 to get 0. So, our problem is solved:

$$-3(-5) = +15$$

Table 1 gives a summary of what we have done so far in this section.

Original Numbers Have	For Example	The Answer Is
Same signs	$3(5) = 15$	Positive
Different signs	$-3(5) = -15$	Negative
Different signs	$3(-5) = -15$	Negative
Same signs	$-3(-5) = 15$	Positive

Table 1

From the examples we have done so far in this section and their summaries in Table 1, we write the following rule for multiplication of positive and negative numbers:

Rule

To multiply any two numbers, we multiply their absolute values.

1. The answer is positive if both the original numbers have the same sign. That is, the product of two numbers with the same sign is positive.
2. The answer is negative if the original two numbers have different signs. The product of two numbers with different signs is negative.

This rule should be memorized. By the time you have finished reading this section and working the problems at the end of the section, you should be fast and accurate at multiplication with positive and negative numbers.

Example 4 Find the following products.

a. $2(4)$ **b.** $-2(-4)$ **c.** $2(-4)$ **d.** $-2(4)$

Solution

a. $2(4) = 8$ Like signs; positive answer

b. $-2(-4) = 8$ Like signs; positive answer

c. $2(-4) = -8$ Unlike signs; negative answer

d. $-2(4) = -8$ Unlike signs; negative answer

Example 5 Simplify $-3(2)(-5)$.

Solution

$$-3(2)(-5) = -6(-5) \quad \text{Multiply } -3 \text{ and } 2 \text{ to get } -6$$
$$= 30$$

Example 6 Use the definition of exponents to expand each expression. Then simplify by multiplying.

a. $(-6)^2$ **b.** -6^2 **c.** $(-4)^3$ **d.** -4^3

Solution

a. $(-6)^2 = (-6)(-6)$ Definition of exponents
$= 36$ Multiply

b. $-6^2 = -6 \cdot 6$ Definition of exponents
$= -36$ Multiply

c. $(-4)^3 = (-4)(-4)(-4)$ Definition of exponents
$= -64$ Multiply

d. $-4^3 = -4 \cdot 4 \cdot 4$ Definition of exponents
$= -64$ Multiply

In Example 6, the base is a negative number in parts a and c, but not in parts b and d. We know this is true because of the use of parentheses.

Example 7 Simplify: $-4 + 5(-6 + 2)$

Solution Simplifying inside the parentheses first, we have

$$-4 + 5(-6 + 2) = -4 + 5(-4) \quad \text{Simplify inside parentheses}$$
$$= -4 + (-20) \quad \text{Multiply}$$
$$= -24 \quad \text{Add}$$

Example 8 Simplify: $-3(2-9)+4(-7-2)$

Solution We begin by subtracting inside the parentheses:

$$\begin{aligned} -3(2-9)+4(-7-2) &= -3(-7)+4(-9) \\ &= 21+(-36) \\ &= -15 \end{aligned}$$

Using Technology Calculators

Here is how we work a similar problem on a calculator. (The [×] key on the first line may, or may not, be necessary. Try your calculator without it and see.)

Scientific Calculator [(] 3 [+/−] [−] 7 [)] [×] [(] 2 [−] 6 [)] [=]

Graphing Calculator [(] [(−)] 3 [−] 7 [)] [(] 2 [−] 6 [)] [ENT]

In either case, the problem you are solving is $(-3-7)(2-6)=40$.

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Write the multiplication problem $3(-5)$ as an addition problem.

B. Write the multiplication problem $2(4)$ as an addition problem.

C. If two numbers have the same sign, then their product will have what sign?

D. If two numbers have different signs, then their product will have what sign?

Problem Set 1.4

Find each of the following products. (Multiply.)

1. $7(-8)$	**2.** $-3(5)$	**3.** $-6(10)$	**4.** $4(-8)$
5. $-7(-8)$	**6.** $-4(-7)$	**7.** $-9(-9)$	**8.** $-6(-3)$
9. $-21(43)$	**10.** $-68(57)$	**11.** $-40(-15)$	**12.** $-80(-30)$
13. $-12(12)$	**14.** $-15(15)$	**15.** $3(-2)(4)$	**16.** $5(-1)(3)$
17. $-4(3)(-2)$	**18.** $-4(5)(-6)$	**19.** $-1(-2)(-3)$	**20.** $-2(-3)(-4)$

Use the definition of exponents to expand each of the following expressions. Then multiply according to the rule for multiplication.

21. **a.** $(-4)^2$ **b.** -4^2	**22.** **a.** $(-5)^2$ **b.** -5^2	**23.** **a.** $(-5)^3$ **b.** -5^3	**24.** **a.** $(-4)^3$ **b.** -4^3
25. **a.** $(-2)^4$ **b.** -2^4	**26.** **a.** $(-1)^4$ **b.** -1^4		

Complete the following tables. Remember, if $x = -5$, then $x^2 = (-5)^2 = 25$.

27.

Number x	Square x^2
-3	
-2	
-1	
0	
1	
2	
3	

28.

Number x	Cube x^3
-3	
-2	
-1	
0	
1	
2	
3	

29.

First Number x	Second Number y	Their Product xy
6	2	
6	1	
6	0	
6	-1	
6	-2	

30.

First Number x	Second Number y	Their Product xy
7	4	
7	2	
7	0	
7	-2	
7	-4	

31.

First Number a	Second Number b	Their Product ab
-5	3	
-5	2	
-5	1	
-5	0	
-5	-1	
-5	-2	
-5	-3	

32.

First Number a	Second Number b	Their Product ab
-9	6	
-9	4	
-9	2	
-9	0	
-9	-2	
-9	-4	
-9	-6	

Use the rule for order of operations along with the rules for addition, subtraction, and multiplication to simplify each of the following expressions.

33. $4(-3 + 2)$

34. $7(-6 + 3)$

35. $-10(-2 - 3)$

36. $-5(-6 - 2)$

37. $-3 + 2(5 - 3)$

38. $-7 + 3(6 - 2)$

39. $-7 + 2[-5 - 9]$

40. $-8 + 3[-4 - 1]$

41. $2(-5) + 3(-4)$

42. $6(-1) + 2(-7)$

43. $3(-2)4 + 3(-2)$

44. $2(-1)(-3) + 4(-6)$

45. $(8 - 3)(2 - 7)$

46. $(9 - 3)(2 - 6)$

47. $(2 - 5)(3 - 6)$

48. $(3 - 7)(2 - 8)$

49. $3(5 - 8) + 4(6 - 7)$

50. $-2(8 - 10) + 3(4 - 9)$

51. $-3(4 - 7) - 2(-3 - 2)$

52. $-5(-2 - 8) - 4(6 - 10)$

53. $3(-2)(6 - 7)$

54. $4(-3)(2 - 5)$

55. Find the product of -3, -2, and -1.

56. Find the product of -7, -1, and 0.

57. What number do you multiply by -3 to get 12?

58. What number do you multiply by -7 to get -21?

59. Subtract -3 from the product of -5 and 4.

60. Subtract 5 from the product of -8 and 1.

Applying the Concepts

For Problems 61 and 62, recall from the introduction to this section that the gain or loss shown for a stock is "per share."

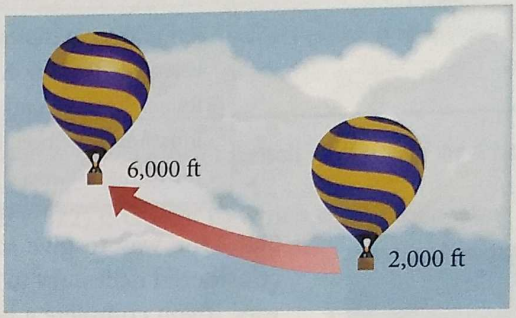
61. Day Trading Larry is buying and selling stock from his home computer. He owns 100 shares of Oracle Corporation and 50 shares of McDonald's Corp. In early January 2013, his stocks had the gain and loss shown in the table below. What was Larry's net gain or loss on those two stocks?

Stock	Number of Shares	Gain/Loss
Oracle	100	-2
McDonald's	50	$+8$

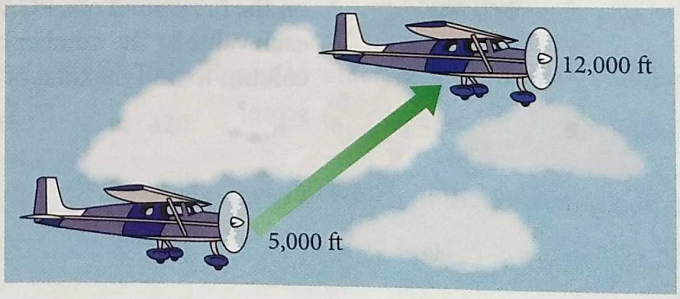
62. Stock Gain/Loss Amy owns stock that she keeps in her retirement account. She owns 200 shares of Apple Computer and 100 shares of Gap Inc. In early January 2013, her stocks had the gain and loss shown in the table below. What was Amy's net gain or loss on those two stocks?

Stock	Number Of Shares	Gain/Loss
Apple	200	+14
Gap	100	−5

63. Temperature Change A hot-air balloon is rising to its cruising altitude. Suppose the air temperature around the balloon drops 4 degrees each time the balloon rises 1,000 feet. What is the net change in air temperature around the balloon as it rises from 2,000 feet to 6,000 feet?



64. Temperature Change A small airplane is rising to its cruising altitude. Suppose the air temperature around the plane drops 4 degrees each time the plane increases its altitude by 1,000 feet. What is the net change in air temperature around the plane as it rises from 5,000 feet to 12,000 feet?



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Baseball Major league baseball has various player awards at the end of each season. Relief pitchers compete for the Rolaids Relief Man Award each year. Points are awarded as follows:

Each win (W) earns 2 points Each loss (L) earns −2 points

Each save (S) earns 3 points Each blown save (BS) earns −2 points

Each tough save* (TS) earns 4 points

The Rolaids points for someone with 4 wins, 2 losses, 8 saves, 1 tough save, and 3 blown saves would be

$$4(2) + 2(-2) + 8(3) + 1(4) + 3(-2) = 26$$

*A tough save occurs when the pitcher enters the game with the potential tying run on base.

Use this information to complete the tables for Problems 65 and 66.

65.

National League

Name, Team	W	L	S	TS	BS	Pts
Craig Kimbrel, Braves	3	1	41	1	3	
Jason Motte, Cardinals	4	5	36	6	7	
Joel Hannahan, Pirates	5	2	36	0	4	
Aroldis Chapman, Reds	5	5	36	2	5	
John Axford, Brewers	5	8	34	1	9	

66.

American League

Name, Team	W	L	S	TS	BS	Pts
Jim Johnson, Orioles	2	1	51	0	3	
Fernando Rodney, Angels	2	2	45	3	2	
Rafael Soriano, Yankees	2	1	39	3	4	
Jonathon Papelbon, Red Sox	5	6	36	2	4	
Joe Nathan, Twins	3	5	37	0	3	

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Golf One way to give scores in golf is in relation to par, the number of strokes considered necessary to complete a hole or course at the expert level. Scoring this way, if you complete a hole in one stroke less than par, your score is -1, which is called a *birdie*. If you shoot 2 under par, your score is -2, which is called an *eagle*. Shooting 1 over par is a score of $+1$, which is a *bogie*. A *double bogie* is 2 over par, and results in a score of $+2$.

67. Sergio Garcia's Scorecard The table below shows the scores Sergio Garcia had on the first round of a PGA tournament. Fill in the last column by multiplying each value by the number of times it occurs. Then add the numbers in the last column to find the total. If par for the course was 72, what was Sergio Garcia's score?

	Value	Number	Product
Eagle	-2	0	
Birdie	-1	7	
Par	0	7	
Bogie	$+1$	3	
Double Bogie	$+2$	1	
		Total:	

68. Karrie Webb's Scorecard The table below shows the scores Karrie Webb had on the final round of an LPGA Standard Register Ping Tournament. Fill in the last column by multiplying each value by the number of times it occurs. Then add the numbers in the last column to find the total. If par for the course was 72, what was Karrie Webb's score?

	Value	Number	Product
Eagle	−2	1	
Birdie	−1	5	
Par	0	8	
Bogie	+1	3	
Double Bogie	+2	1	
		Total:	

Estimating

Work Problems 69–76 mentally, without pencil and paper or a calculator.

69. The product $-32(-522)$ is closest to which of the following numbers?

a. 15,000 **b.** −500 **c.** −1,500 **d.** −15,000

70. The product $32(-522)$ is closest to which of the following numbers?

a. 15,000 **b.** −500 **c.** −1,500 **d.** −15,000

71. The product $-47(470)$ is closest to which of the following numbers?

a. 25,000 **b.** 420 **c.** −2,500 **d.** −25,000

72. The product $-47(-470)$ is closest to which of the following numbers?

a. 25,000 **b.** 420 **c.** −2,500 **d.** −25,000

73. The product $-222(-987)$ is closest to which of the following numbers?

a. 200,000 **b.** 800 **c.** −800 **d.** −1,200

74. The sum $-222 + (-987)$ is closest to which of the following numbers?

a. 200,000 **b.** 800 **c.** −800 **d.** −1,200

75. The difference $-222 - (-987)$ is closest to which of the following numbers?

a. 200,000 **b.** 800 **c.** −800 **d.** −1,200

76. The difference $-222 - 987$ is closest to which of the following numbers?

a. 200,000 **b.** 800 **c.** −800 **d.** −1,200

Getting Ready for the Next Section

Perform the indicated operations.

77. $12 \div 4$

78. $32 \div 4$

79. $\dfrac{20}{5}$

80. $\dfrac{30}{5}$

81. $12 - 17$

82. $-15 + 5(-4)$

83. $\dfrac{6(3)}{2}$

84. $\dfrac{8(5)}{4}$

85. $80 \div 10 \div 2$

86. $80 \div 2 \div 10$

87. $\dfrac{15 + 5(4)}{17 - 12}$

88. $\dfrac{20 + 6(2)}{11 - 7}$

89. $4(10^2) + 20 \div 4$

90. $3(4^2) + 10 \div 5$