Subtraction with Negative Numbers

1.3

© Elena Elisseeva/iStockPhoto

Earlier in this chapter we asked how we would represent the final balance in a checkbook if the original balance was \$20 and we wrote a check for \$30. We decided that the final balance would be −\$10. We can summarize the whole situation with subtraction:

$$\$20 - \$30 = -\$10$$

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (-)	DEPOSIT/CREDIT (+)	BALANCE
					\$20 00
1501	9/15	Campus Bookstore	\$30 00		-\$10 00

From this we see that subtracting 30 from 20 gives us −10. Another example that gives the same answer but involves addition is this:

$$20 + (-30) = -10$$

From the two examples above, we find that subtracting 30 gives the same result as adding −30. We use this kind of reasoning to give a definition for subtraction that will allow us to use the rules we developed for addition to do our subtraction problems. Here is that definition:

Subtraction

If a and b represent any two numbers, then it is always true that

$$a - b = a + (-b)$$

To subtract b — Add its opposite, $-b$

In words Subtracting a number is equivalent to adding its opposite.

Note This definition of subtraction may seem a little strange at first. In the example shown here, you will notice that using the definition (to convert to "5 plus the opposite of 2") gives us the same result we are used to getting with subtraction (5 minus 2). As we progress further into the section, we will use the definition to subtract numbers we haven't been able to subtract before.

Let's see if this definition conflicts with what we already know to be true about subtraction.

From previous experience we know that

$$5 - 2 = 3$$

We can get the same answer by using the definition we just gave for subtraction. Instead of subtracting 2, we can add its opposite, −2. Here is how it looks:

$$5 - 2 = 5 + (-2) \qquad \text{Change subtraction to addition of the opposite}$$

$$= 3 \qquad \text{Apply the rule for addition of positive and negative numbers}$$

The result is the same whether we use our previous knowledge of subtraction or the new definition. The new definition is essential when the problems begin to get more complicated.

Note A real-life analogy to Example 1 would be: "If the temperature were 7° below 0 and then it dropped another 2°, what would the temperature be then?"

Example 1 Subtract: $-7 - 2$

Solution We have never subtracted a positive number from a negative number before. We must apply our definition of subtraction:

$$-7 - 2 = -7 + (-2)$$ Instead of subtracting 2, we add its opposite, -2

$$= -9$$ Apply the rule for addition

VIDEO EXAMPLES

SECTION 1.3

Example 2 Subtract: $12 - (-6)$

Solution The first $-$ sign is read "subtract," and the second one is read "negative." The problem in words is "12 subtract negative 6." We can use the definition of subtraction to change this to the addition of positive 6:

$$12 - (-6) = 12 + 6$$ Subtracting -6 is equivalent to adding $+6$

$$= 18$$ Addition

Example 3 The following table shows the relationship between subtraction and addition:

Subtraction	Addition of the opposite	Answer
$7 - 9$	$7 + (-9)$	-2
$-7 - 9$	$-7 + (-9)$	-16
$7 - (-9)$	$7 + 9$	16
$-7 - (-9)$	$-7 + 9$	2
$15 - 10$	$15 + (-10)$	5
$-15 - 10$	$-15 + (-10)$	-25
$15 - (-10)$	$15 + 10$	25
$-15 - (-10)$	$-15 + 10$	-5

Examples 1–3 illustrate all the possible combinations of subtraction with positive and negative numbers. There are no new rules for subtraction. We apply the definition to change each subtraction problem into an equivalent addition problem. The rule for addition can then be used to obtain the correct answer.

Example 4 Combine: $-3 + 6 - 2$

Solution The first step is to change subtraction to addition of the opposite. After that has been done, we add left to right.

$$-3 + 6 - 2 = -3 + 6 + (-2)$$ Subtracting 2 is equivalent to adding -2

$$= 3 + (-2)$$ Add left to right

$$= 1$$

Example 5 Subtract 3 from -5.

Solution Subtracting 3 is equivalent to adding -3.

$$-5 - 3 = -5 + (-3) = -8$$

Subtracting 3 from -5 gives us -8.

Applying the Concepts

© Glenn Frank/iStockPhoto

Example 6 Many of the planes used by the United States during World War II were not pressurized or sealed from outside air. As a result, the temperature inside these planes was the same as the surrounding air temperature outside. Suppose the temperature inside a B-17 Flying Fortress is 50 °F at takeoff and then drops to −30 °F when the plane reaches its cruising altitude of 28,000 feet. Find the difference in temperature inside this plane at takeoff and at 28,000 feet.

Solution The temperature at takeoff is 50 °F, whereas the temperature at 28,000 feet is −30 °F. To find the difference we subtract, with the numbers in the same order as they are given in the problem:

$$50 - (-30) = 50 + 30 = 80$$

The difference in temperature is 80 °F.

Subtraction and Taking Away

Some people may believe that the answer to $-5 - 9$ should be -4 or 4, not -14. If you find this happening, you are probably thinking of subtraction in terms of taking one number away from another. Thinking of subtraction in this way works well with positive numbers if you always subtract the smaller number from the larger. In algebra, however, we encounter many situations other than this. The definition of subtraction, that $a - b = a + (-b)$, clearly indicates the correct way to use subtraction. That is, when working subtraction problems, you should think "addition of the opposite," not "taking one number away from another."

Note When working subtraction problems on a calculator, there is no need to rewrite subtraction as "addition of the opposite."

Using Technology Calculator

Here is how we work the subtraction problem shown in Example 1 on a calculator.

Scientific Calculator 7 [+/−] [−] 2 [=]

Graphing Calculator [(−)] 7 [−] 2 [ENT]

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Write the subtraction problem $5 - 3$ as an equivalent addition problem.

B. Explain the process you would use to subtract 2 from -7.

C. Write an addition problem that is equivalent to the subtraction problem $-20 - (-30)$.

D. To find the difference of -7 and -4 we subtract what number from -7? What is the associated addition problem?

Problem Set 1.3

Subtract.

1. $7 - 5$	**2.** $5 - 7$	**3.** $8 - 6$	**4.** $6 - 8$
5. $-3 - 5$	**6.** $-5 - 3$	**7.** $-4 - 1$	**8.** $-1 - 4$
9. $5 - (-2)$	**10.** $2 - (-5)$	**11.** $3 - (-9)$	**12.** $9 - (-3)$
13. $-4 - (-7)$	**14.** $-7 - (-4)$	**15.** $-10 - (-3)$	**16.** $-3 - (-10)$
17. $15 - 18$	**18.** $20 - 32$	**19.** $100 - 113$	**20.** $121 - 21$
21. $-30 - 20$	**22.** $-50 - 60$	**23.** $-79 - 21$	**24.** $-86 - 31$
25. $156 - (-243)$	**26.** $292 - (-841)$	**27.** $-35 - (-14)$	**28.** $-29 - (-4)$

Complete the following tables.

29.

First Number x	Second Number y	the Difference of x and y $x - y$
8	6	
8	7	
8	8	
8	9	
8	10	

30.

First Number x	Second Number y	the Difference of x and y $x - y$
10	12	
10	11	
10	10	
10	9	
10	8	

31.

First Number x	Second Number y	the Difference of x and y $x - y$
8	-6	
8	-7	
8	-8	
8	-9	
8	-10	

32.

First Number x	Second Number y	the Difference of x and y $x - y$
-10	-12	
-10	-11	
-10	-10	
-10	-9	
-10	-8	

Simplify as much as possible by first changing all subtractions to addition of the opposite and then adding left to right.

33. $4 - 5 - 6$	**34.** $7 - 3 - 2$
35. $-8 + 3 - 4$	**36.** $-10 - 1 + 16$
37. $-8 - 4 - 2$	**38.** $-7 - 3 - 6$
39. $33 - (-22) - 66$	**40.** $44 - (-11) + 55$
41. $-900 + 400 - (-100)$	**42.** $-300 + 600 - (-200)$

43. Subtract -6 from 5.

44. Subtract 8 from -2.

45. Find the difference of -5 and -1.

46. Find the difference of -7 and -3.

47. Subtract -4 from the sum of -8 and 12.

48. Subtract -7 from the sum of 7 and -12.

49. What number do you subtract from -3 to get -9?

50. What number do you subtract from 5 to get 8?

Estimating

Work Problems 51–60 mentally, without pencil and paper or a calculator.

51. The answer to the problem $52 - 49$ is closest to which of the following numbers?

    a. 100 **b.** 0 **c.** -100

52. The answer to the problem $-52 - 49$ is closest to which of the following numbers?

    a. 100 **b.** 0 **c.** -100

53. The answer to the problem $52 - (-49)$ is closest to which of the following numbers?

    a. 100 **b.** 0 **c.** -100

54. The answer to the problem $-52 - (-49)$ is closest to which of the following numbers?

    a. 100 **b.** 0 **c.** -100

55. Is the difference $-161 - (-62)$ closer to -200 or -100?

56. Is the difference $-553 - 50$ closer to -600 or -500?

57. The difference of 37 and 61 is closest to which of the following numbers?

    a. -100 **b.** -20 **c.** 20 **d.** 100

58. The difference of 37 and -61 is closest to which of the following numbers?

    a. -100 **b.** -20 **c.** 20 **d.** 100

59. The difference of -37 and 61 is closest to which of the following numbers?

    a. -100 **b.** -20 **c.** 20 **d.** 100

60. The difference of -37 and -61 is closest to which of the following numbers?

    a. -100 **b.** -20 **c.** 20 **d.** 100

Applying the Concepts

61. Temperature On Monday the temperature reached a high of 28° above 0. That night it dropped to 16° below 0. What is the difference between the high and the low temperatures for Monday?

© Baris Simsek/iStockPhoto

62. Gambling A gambler loses $35 playing roulette one night and then loses another $25 the next night. Express this situation with numbers. How much did the gambler lose?

63. Submarine Depth A submarine is located 450 feet below sea level. If it descends 125 feet, what is its new position?

64. Scuba Diving Trevor and Mike are scuba diving while on vacation in Hawaii. Mike descends to a point 17 feet below the ocean surface. Trevor dives to a point 33 feet below the surface. Represent their depths with negative numbers and use subtraction to find how much deeper Trevor's dive was than Mike's.

65. Highest and Lowest Points The world's tallest mountain is Mount Everest, with an elevation of 29,035 feet above sea level. The shoreline of the Dead Sea is said to be the lowest dry land point on earth, at about 1,355 feet below sea level. What is the difference between these two elevations?

66. Checkbook Balance Susan has a balance of $572 in her checking account when she writes a check for $435 to pay the rent. Then she writes another check for $172 for textbooks. Write a subtraction problem that gives the new balance in her checking account. What is the new balance in her checking account?

Repeated below is the table of wind chill temperatures that we used previously. Use it for Problems 67–70.

	Air Temperature (°F)							
Wind speed	30	25	20	15	10	5	0	−5
10 mph	16	10	3	−3	−9	−15	−22	−27
15 mph	9	2	−5	−11	−18	−25	−31	−38
20 mph	4	−3	−10	−17	−24	−31	−39	−46
25 mph	1	−7	−15	−22	−29	−36	−44	−51
30 mph	−2	−10	−18	−25	−33	−41	−49	−56

67. Wind Chill If the temperature outside is 15 °F, what is the difference in wind chill temperature between a 15-mile-per-hour wind and a 25-mile-per-hour wind?

68. Wind Chill If the temperature outside is 0 °F, what is the difference in wind chill temperature between a 15-mile-per-hour wind and a 25-mile-per-hour wind?

69. Wind Chill Find the difference in temperature between a day in which the air temperature is 20 °F and the wind is blowing at 10 miles per hour and a day in which the air temperature is 10 °F and the wind is blowing at 20 miles per hour.

70. Wind Chill Find the difference in temperature between a day in which the air temperature is 0 °F and the wind is blowing at 10 miles per hour and a day in which the air temperature is −5 °F and the wind is blowing at 20 miles per hour.

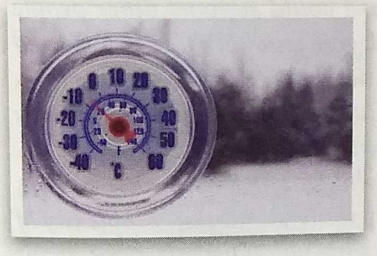
Use the tables below to work Problems 71–74.

Record low temperatures for Lake Placid, New York

Month	Temperature
January	−36 °F
February	−37 °F
March	−30 °F
April	−5 °F
May	19 °F
June	22 °F
July	31 °F
August	27 °F
September	19 °F
October	11 °F
November	−11 °F
December	−31 °F

Record high temperatures for Lake Placid, New York

Month	Temperature
January	62 °F
February	62 °F
March	78 °F
April	86 °F
May	90 °F
June	93 °F
July	97 °F
August	94 °F
September	94 °F
October	87 °F
November	74 °F
December	63 °F



© benoitb/iStockPhoto

71. Temperature Difference Find the difference between the record high temperature and the record low temperature for the month of December.

72. Temperature Difference Find the difference between the record high temperature and the record low temperature for the month of March.

73. Temperature Difference Find the difference between the record low temperatures of March and December.

74. Temperature Difference Find the difference between the record high temperatures of March and December.

Getting Ready for the Next Section

Perform the indicated operations.

75. $(-5) + (-5) + (-5)$

76. $(-3) + (-3) + (-3) + (-3) + (-3)$

77. $3(2)(5)$

78. $5(2)(4)$

79. 6^2

80. 8^2

81. 4^3

82. 3^3

83. $3(9 - 2) + 4(7 - 2)$

84. $2(5 - 3) - 7(4 - 2)$

85. $(3 + 7)(6 - 2)$

86. $(6 + 1)(9 - 4)$