Addition with Negative Numbers 1.2

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Suppose you are in Las Vegas playing blackjack and you lose \$3 on the first hand and then you lose \$5 on the next hand. If you represent winning with positive numbers and losing with negative numbers, how will you represent the results from your first two hands? Since you lost \$3 and \$5 for a total of \$8, one way to represent the situation is with addition of negative numbers:

$$(-\$3) + (-\$5) = -\$8$$

From this example we see that the sum of two negative numbers is a negative number. To generalize addition of positive and negative numbers, we can use the number line.

We can think of each number on the number line as having two characteristics: (1) a *distance* from 0 (absolute value) and (2) a *direction* from 0 (positive or negative). The distance from 0 is represented by the numerical part of the number (like the 5 in the number -5), and its direction is represented by the $+$ or $-$ sign in front of the number.

We can visualize addition of numbers on the number line by thinking in terms of distance and direction from 0. Let's begin with a simple problem we know the answer to. We interpret the sum $3 + 5$ on the number line as follows:

1. The first number is 3, which tells us "start at the origin, and move 3 units in the positive direction."
2. The $+$ sign is read "and then move."
3. The 5 means "5 units in the positive direction."

Note This method of adding numbers may seem a little complicated at first, but it will allow us to add numbers we couldn't otherwise add.

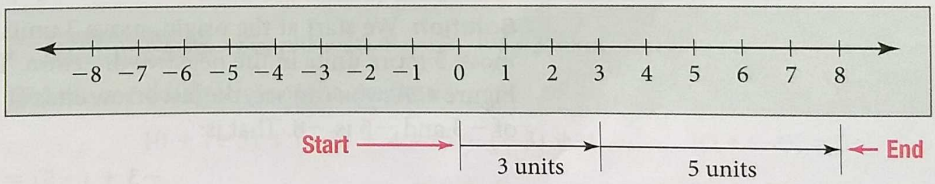

Figure 1

Figure 1 shows these steps. To summarize, $3 + 5$ means to start at the origin (0), move 3 units in the *positive* direction, and then move 5 units in the *positive* direction. We end up at 8, which is the sum we are looking for: $3 + 5 = 8$.

VIDEO EXAMPLES

SECTION 1.2

Example 1 Add $3 + (-5)$ using the number line.

Solution We start at the origin, move 3 units in the positive direction, and then move 5 units in the negative direction, as shown in Figure 2. The last arrow ends at -2, which must be the sum of 3 and -5. That is:

$$3 + (-5) = -2$$

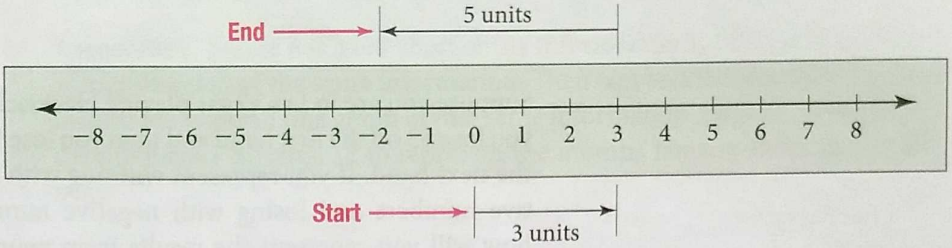

Figure 2

Example 2 Add $-3 + 5$ using the number line.

Solution We start at the origin, move 3 units in the negative direction, and then move 5 units in the positive direction, as shown in Figure 3. We end up at 2, which is the sum of -3 and 5. That is:

$$-3 + 5 = 2$$

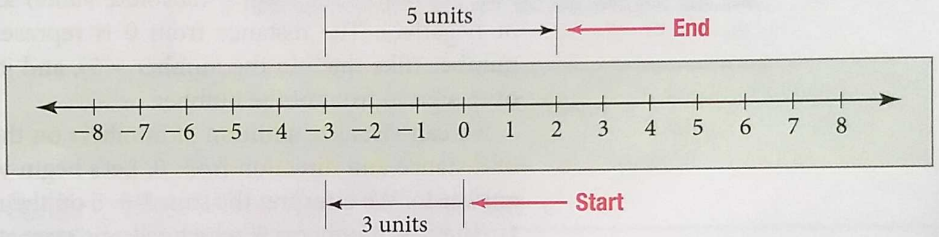

Figure 3

Example 3 Add $-3 + (-5)$ using the number line.

Solution We start at the origin, move 3 units in the negative direction, and then move 5 more units in the negative direction. This is shown on the number line in Figure 4. As you can see, the last arrow ends at -8. We must conclude that the sum of -3 and -5 is -8. That is:

$$-3 + (-5) = -8$$

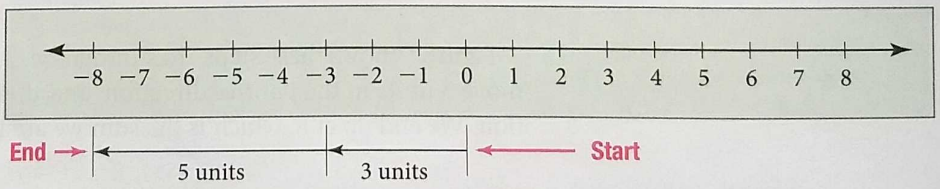

Figure 4

Adding numbers on the number line as we have done in these first three examples gives us a way of visualizing addition of positive and negative numbers. We want to be able to write a rule for addition of positive and negative numbers that doesn't involve the number line. The number line is a way of justifying the rule we will write. Here is a summary of the results we have so far:

$$3 + 5 = 8 \qquad -3 + 5 = 2$$
$$3 + (-5) = -2 \qquad -3 + (-5) = -8$$

Looking over these results, we write the following rule for adding any two numbers:

Note This rule covers all possible addition problems involving positive and negative numbers. You must memorize it. After you have worked some problems, the rule will seem almost automatic.

Rule for Adding Any Two Numbers

1. To add two numbers with the same sign: Simply add their absolute values, and use the common sign. If both numbers are positive, the answer is positive. If both numbers are negative, the answer is negative.
2. To add two numbers with different signs: Subtract the smaller absolute value from the larger absolute value. The answer will have the sign of the number with the larger absolute value.

The following examples show how the rule is used. You will find that the rule for addition is consistent with all the results obtained using the number line.

Example 4 Add all combinations of positive and negative 10 and 15.

Solution

$$\begin{aligned} 10 + 15 &= 25 \\ 10 + (-15) &= -5 \\ -10 + 15 &= 5 \\ -10 + (-15) &= -25 \end{aligned}$$

Notice that when we add two numbers with the same sign, the answer also has that sign. When the signs are not the same, the answer has the sign of the number with the larger absolute value.

Once you have become familiar with the rule for adding positive and negative numbers, you can apply it to more complicated sums.

Example 5 Simplify: $10 + (-5) + (-3) + 4$

Solution Adding left to right, we have:

$$\begin{aligned} 10 + (-5) + (-3) + 4 &= 5 + (-3) + 4 \qquad 10 + (-5) = 5 \\ &= 2 + 4 \qquad 5 + (-3) = 2 \\ &= 6 \end{aligned}$$

Example 6 Simplify: $[-3 + (-10)] + [8 + (-2)]$

Solution We begin by adding the numbers inside the brackets.

$$\begin{aligned} [-3 + (-10)] + [8 + (-2)] &= [-13] + [6] \\ &= -7 \end{aligned}$$

Using Technology Calculators

Here is how we work the addition problem in Example 3 on a calculator:

Scientific Calculator 3 [+/−] [+] 5 [+/−] [=]

Graphing Calculator [(−)] 3 [+] [(−)] 5 [ENT]

Problem Set 1.2

Draw a number line from -10 to $+10$ and use it to add the following numbers.

1. $2 + 3$ **2.** $2 + (-3)$ **3.** $-2 + 3$ **4.** $-2 + (-3)$

5. $5 + (-7)$ **6.** $-5 + 7$ **7.** $-4 + (-2)$ **8.** $-8 + (-2)$

9. $10 + (-6)$ **10.** $-9 + 3$ **11.** $7 + (-3)$ **12.** $-7 + 3$

13. $-4 + (-5)$ **14.** $-2 + (-7)$

Combine the following by using the rule for addition of positive and negative numbers. (Your goal is to be fast and accurate at addition, with the latter being more important.)

15. $7 + 8$ **16.** $9 + 12$ **17.** $5 + (-8)$ **18.** $4 + (-11)$

19. $-6 + (-5)$ **20.** $-7 + (-2)$ **21.** $-10 + 3$ **22.** $-14 + 7$

23. $-1 + (-2)$ **24.** $-5 + (-4)$ **25.** $-11 + (-5)$ **26.** $-16 + (-10)$

27. $4 + (-12)$ **28.** $9 + (-1)$ **29.** $-85 + (-42)$ **30.** $-96 + (-31)$

31. $-121 + 170$ **32.** $-130 + 158$ **33.** $-375 + 409$ **34.** $-765 + 213$

Complete the following tables.

35.

First Number a	Second Number b	Their Sum $a + b$
5	-3	
5	-4	
5	-5	
5	-6	
5	-7	

36.

First Number a	Second Number b	Their Sum $a + b$
-5	3	
-5	4	
-5	5	
-5	6	
-5	7	

37.

First Number x	Second Number y	Their Sum $x + y$
-5	-3	
-5	-4	
-5	-5	
-5	-6	
-5	-7	

38.

First Number x	Second Number y	Their Sum $x + y$
30	-20	
-30	20	
-30	-20	
30	20	
-30	0	

Add the following numbers left to right.

39. $10 + (-18) + 4$ **40.** $-2 + 4 + (-6)$

41. $24 + (-6) + (-8)$ **42.** $35 + (-5) + (-30)$

43. $-201 + (-143) + (-101)$ **44.** $-27 + (-56) + (-89)$

45. $-321 + 752 + (-324)$ **46.** $-571 + 437 + (-502)$

47. $-8 + 3 + (-5) + 9$ **48.** $-9 + 2 + (-10) + 3$

49. $-2 + (-5) + (-6) + (-7)$

50. $-8 + (-3) + (-4) + (-7)$

51. $15 + (-30) + 18 + (-20)$

52. $20 + (-15) + 30 + (-18)$

53. $-78 + (-42) + 57 + 13$

54. $-89 + (-51) + 65 + 17$

Use the rule for order of operations to simplify each of the following.

55. $(-8 + 5) + (-6 + 2)$

56. $(-3 + 1) + (-9 + 4)$

57. $(-10 + 4) + (-3 + 12)$

58. $(-11 + 5) + (-3 + 2)$

59. $20 + (-30 + 50) + 10$

60. $30 + (-40 + 20) + 50$

61. $108 + (-456 + 275)$

62. $106 + (-512 + 318)$

63. $[5 + (-8)] + [3 + (-11)]$

64. $[8 + (-2)] + [5 + (-7)]$

65. $[57 + (-35)] + [19 + (-24)]$

66. $[63 + (-27)] + [18 + (-24)]$

67. Find the sum of -8, -10, and -3.

68. Find the sum of -4, 17, and -6.

69. What number do you add to 8 to get 3?

70. What number do you add to 10 to get 4?

71. What number do you add to -3 to get -7?

72. What number do you add to -5 to get -8?

73. What number do you add to -4 to get 3?

74. What number do you add to -7 to get 2?

75. If the sum of -3 and 5 is increased by 8, what number results?

76. If the sum of -9 and -2 is increased by 10, what number results?

Estimating

Work Problems 77–84 mentally, without pencil and paper or a calculator.

77. The answer to the problem $251 + 249$ is closest to which of the following numbers?

a. 500 **b.** 0 **c.** -500

78. The answer to the problem $251 + (-249)$ is closest to which of the following numbers?

a. 500 **b.** 0 **c.** -500

79. The answer to the problem $-251 + 249$ is closest to which of the following numbers?

a. 500 **b.** 0 **c.** -500

80. The answer to the problem $-251 + (-249)$ is closest to which of the following numbers?

a. 500 **b.** 0 **c.** -500

81. The sum of 77 and 22 is closest to which of the following numbers?

a. -100 **b.** -60 **c.** 60 **d.** 100

82. The sum of -77 and 22 is closest to which of the following numbers?

a. -100 **b.** -60 **c.** 60 **d.** 100

83. The sum of 77 and -22 is closest to which of the following numbers?

a. -100 **b.** -60 **c.** 60 **d.** 100

84. The sum of -77 and -22 is closest to which of the following numbers?

a. -100 **b.** -60 **c.** 60 **d.** 100

Applying the Concepts

85. Checkbook Balance Ethan has a balance of $-\$40$ in his checkbook. If he deposits \$100 and then writes a check for \$50, what is the new balance in his checkbook?

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (-)	DEPOSIT/CREDIT (+)	BALANCE
					-$40 00
	9/20	Deposit		$100 00	
1502	9/21	Vons Market	$50 00		

86. Checkbook Balance Kendra has a balance of $-\$20$ in her checkbook. If she deposits \$45 and then writes a check for \$15, what is the new balance in her checkbook?

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87. Gambling While gambling in Las Vegas, a person wins \$74 playing blackjack and then loses \$141 on roulette. Use positive and negative numbers to write this situation in symbols. Then give the person's net loss or gain.

88. Gambling While playing blackjack, a person loses \$17 on his first hand, then wins \$14, and then loses \$21. Write this situation using positive and negative numbers and addition; then simplify.

89. Stock Gain/Loss Suppose a certain stock gains 3 points on the stock exchange on Monday and then loses 5 points on Tuesday. Express the situation using positive and negative numbers, and then give the net gain or loss of the stock for this 2-day period.

90. Stock Gain/Loss A stock gains 2 points on Wednesday, then loses 1 on Thursday, and gains 3 on Friday. Use positive and negative numbers and addition to write this situation in symbols, and then simplify.

91. Distance The distance between two numbers on the number line is 10. If one of the numbers is 3, what are the two possibilities for the other number?

92. Distance The distance between two numbers on the number line is 8. If one of the numbers is -5, what are the two possibilities for the other number?

Getting Ready for the Next Section

Give the opposite of each number.

93. 2 **94.** 3 **95.** -6 **96.** -30

97. Subtract 3 from 5.

98. Subtract 2 from 8.

99. Find the difference of 7 and 4.

100. Find the difference of 8 and 6.