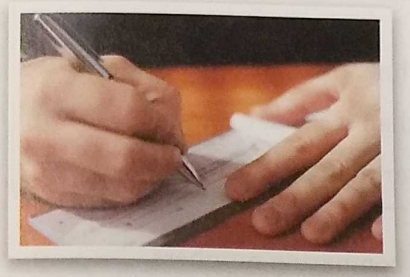
Positive and Negative Numbers

1.1



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Suppose you have a balance of \$20 in your checkbook and then write a check for \$30. You are now overdrawn by \$10. How will you write this new balance? One way is with a negative number. You could write the balance as −\$10, which is a negative number.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT

NUMBER	DATE	DESCRIPTION OF TRANSACTION	PAYMENT/DEBIT (−)	DEPOSIT/CREDIT (+)	BALANCE
					\$20 00
1501	9/15	Campus Bookstore	\$30 00		−\$10 00

Negative numbers can be used to describe other situations as well—for instance, temperature below zero and distance below sea level.

To see the relationship between negative and positive numbers, we can extend the number line as shown in Figure 1. We first draw a straight line and label a convenient point with 0. This is called the **origin**, and it is usually in the middle of the line. We then label positive numbers to the right (as we have done previously), and negative numbers to the left.

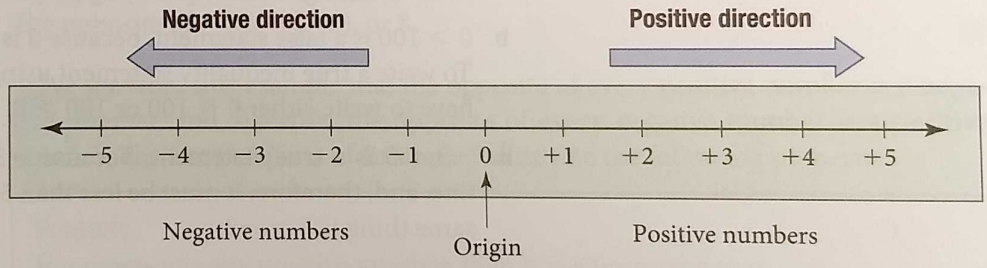

Figure 1

Note A number, other than 0, with no sign (+ or −) in front of it is assumed to be positive. That is, 5 = +5.

The numbers increase going from left to right. If we move to the right, we are moving in the positive direction. If we move to the left, we are moving in the negative direction. *Any number to the left of another number is considered to be smaller than the number to its right.*

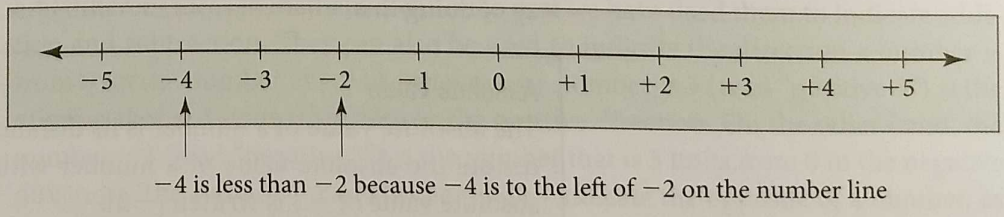

Figure 2

We see from the number line on the previous page that every negative number is less than every positive number.

Notation

If a and b are any two numbers on the number line, then

$a < b$ is read "a is less than b"

$a > b$ is read "a is greater than b"

In algebra we can use inequality symbols when comparing numbers.

As you can see, the inequality symbols always point to the smaller of the two numbers being compared. Here are some examples that illustrate how we use the inequality symbols.

VIDEO EXAMPLES

SECTION 1.1

Example 1 Explain the meaning of each expression.

a. $3 < 5$ **b.** $0 > 100$ **c.** $-3 < 5$ **d.** $-5 < -2$

Solution

a. $3 < 5$ is read "3 is less than 5." Note that it would also be correct to write $5 > 3$. Both statements, "3 is less than 5" and "5 is greater than 3," have the same meaning. The inequality symbols always point to the smaller number.

b. $0 > 100$ is a false statement, because 0 is less than 100, not greater than 100. To write a true inequality statement using the numbers 0 and 100, we would have to write either $0 < 100$ or $100 > 0$.

c. $-3 < 5$ is a true statement, because -3 is to the left of 5 on the number line, and, therefore, it must be less than 5. Another statement that means the same thing is $5 > -3$.

d. $-5 < -2$ is a true statement, because -5 is to the left of -2 on the number line, meaning that -5 is less than -2. Both statements $-5 < -2$ and $-2 > -5$ have the same meaning; they both say that -5 is a smaller number than -2.

It is sometimes convenient to talk about only the numerical part of a number and disregard the sign (+ or −) in front of it. The following definition gives us a way of doing this.

Absolute Value

The **absolute value** of a number is its distance from 0 on the number line. We denote the absolute value of a number with vertical lines. For example, the absolute value of -3 is written $|-3|$.

The absolute value of a number is never negative because it is a distance, and a distance is always measured in positive units (unless it happens to be 0).

Example 2 Simplify each expression.

a. $|5|$ **b.** $|-3|$ **c.** $|-7|$

Solution

a. $|5| = 5$ The number 5 is 5 units from 0.

b. $|-3| = 3$ The number -3 is 3 units from 0.

c. $|-7| = 7$ The number -7 is 7 units from 0.

Note In some books *opposites* are called *additive inverses*.

Opposites

Two numbers that are the same distance from 0 but in opposite directions from 0 are called **opposites.** The notation for the opposite of a is $-a$.

Example 3 Give the opposite of each of the following numbers:

$$5, 7, 1, -5, -8$$

Solution

The opposite of 5 is -5.
The opposite of 7 is -7.
The opposite of 1 is -1.
The opposite of -5 is $-(-5)$, or 5.
The opposite of -8 is $-(-8)$, or 8.

We see from this example that the opposite of every positive number is a negative number, and, likewise, the opposite of every negative number is a positive number. The last two parts of Example 3 illustrate the following property:

Property

If a represents any positive number, then it is always true that

$$-(-a) = a$$

In other words, this property states that the opposite of a negative number is a positive number.

It should be evident now that the symbols $+$ and $-$ can be used to indicate several different ideas in mathematics. In the past we have used them to indicate addition and subtraction. They can also be used to indicate the direction a number is from 0 on the number line. For instance, the number $+3$ (read "positive 3") is the number that is 3 units from zero in the positive direction. On the other hand, the number -3 (read "negative 3") is the number that is 3 units from 0 in the negative direction. The symbol $-$ can also be used to indicate the opposite of a number, as in $-(-2) = 2$. The interpretation of the symbols $+$ and $-$ depends on the situation in which they are used. For example:

$3 + 5$	The $+$ sign indicates addition.
$+4$	The $+$ sign is read "positive" 4.
$7 - 2$	The $-$ sign indicates subtraction.
-7	The $-$ sign is read "negative" 7.
$-(-5)$	The first $-$ sign is read "the opposite of." The second $-$ sign is read "negative" 5.

This may seem confusing at first, but as you work through the problems in this chapter you will get used to the different interpretations of the symbols + and −.

We should mention here that the set of whole numbers along with their opposites forms the set of **integers**. That is:

$$\text{Integers} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Scatter Diagrams and Line Graphs

The information in the table and bar chart on the first page of this chapter can be visualized with a **scatter diagram** and **line graph** as well. Figure 3 is a scatter diagram of the information in Table 1. We use dots instead of bars to show the low temperatures for each month. Figure 4 is called a **line graph**. It is constructed by taking the dots in Figure 3 and connecting each one to the next with a straight line.

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Temp (°F)	−50	−44	−32	−5	5	19	24	18	8	2	−27	−49

Table 1

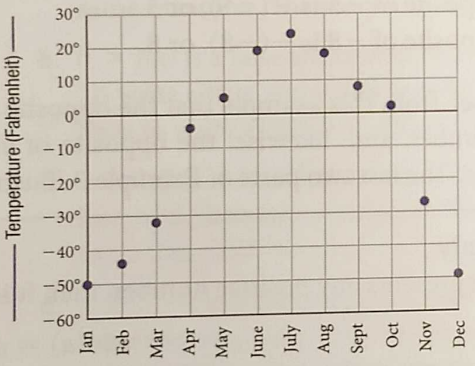

Figure 3

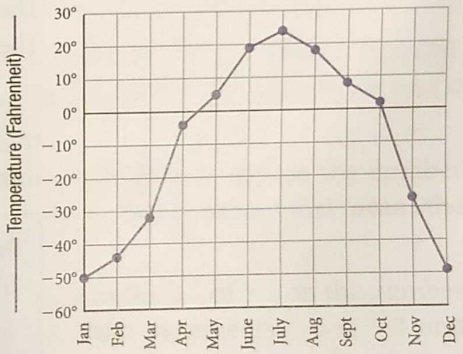

Figure 4

Getting Ready for Class

After reading through the preceding section, respond in your own words and in complete sentences.

A. Write the statement "3 is less than 5" in symbols.

B. What is the absolute value of a number?

C. Describe what we mean by numbers that are "opposites" of each other.

D. If you locate two different numbers on the real number line, which one will be the smaller number?

Problem Set 1.1

Write each of the following in words.

1. $4 < 7$ **2.** $0 < 10$ **3.** $5 > -2$ **4.** $8 > -8$

5. $-10 < -3$ **6.** $-20 < -5$ **7.** $0 > -4$ **8.** $0 > -100$

Write each of the following in symbols.

9. 30 is greater than -30.

10. -30 is less than 30.

11. -10 is less than 0.

12. 0 is greater than -10.

13. -3 is greater than -15.

14. -15 is less than -3.

Place either $<$ or $>$ between each of the following pairs of numbers so that the resulting statement is true.

15. 3 7 **16.** 17 0 **17.** 7 -5 **18.** 2 -13

19. -6 0 **20.** -14 0 **21.** -12 -2 **22.** -20 -1

23. -1 -3 **24.** -6 5 **25.** -75 25 **26.** -3 -1

27. -100 -10 **28.** -4 -40 **29.** -3 $|6|$ **30.** $|8|$ -2

31. 15 $|-4|$ **32.** 20 $|-6|$ **33.** $|-2|$ $|-7|$ **34.** $|-3|$ $|-1|$

Find each of the following absolute values.

35. $|2|$ **36.** $|7|$ **37.** $|100|$ **38.** $|10{,}000|$

39. $|-8|$ **40.** $|-9|$ **41.** $|-231|$ **42.** $|-457|$

43. $|-3|$ **44.** $|-1|$ **45.** $|-200|$ **46.** $|-350|$

47. $|8|$ **48.** $|9|$ **49.** $|231|$ **50.** $|457|$

Give the opposite of each of the following numbers.

51. 3 **52.** -5 **53.** -2 **54.** 15 **55.** 75 **56.** -32

57. 0 **58.** 1 **59.** -123 **60.** -345 **61.** 700 **62.** 100

Simplify each of the following.

63. $-(-2)$ **64.** $-(-5)$ **65.** $-(-8)$ **66.** $-(-3)$

67. $-|-2|$ **68.** $-|-5|$ **69.** $-|-8|$ **70.** $-|-3|$

71. What number is its own opposite?

72. Is $|a| = a$ always a true statement?

73. If n is a negative number, is $-n$ positive or negative?

74. If n is a positive number, is $-n$ positive or negative?

Estimating

Work Problems 75–80 mentally, without pencil and paper or a calculator.

75. Is -60 closer to 0 or -100?

76. Is -20 closer to 0 or -30?

77. Is -10 closer to -20 or 20?

78. Is -20 closer to -40 or 10?

79. Is -362 closer to -360 or -370?

80. Is -368 closer to -360 or -370?

Applying the Concepts

81. Temperature and Altitude Yamina is flying from Phoenix to San Francisco on a Boeing 737 jet. When the plane reaches an altitude of 33,000 feet, the temperature outside the plane is 61 degrees below zero Fahrenheit. Represent this temperature with a negative number. If the temperature outside the plane gets warmer by 10 degrees, what will the new temperature be?

82. Temperature Change At 11:00 in the morning in Superior, Wisconsin, Jim notices the temperature is 15 degrees below zero Fahrenheit. Write this temperature as a negative number. At noon it has warmed up by 8 degrees. What is the temperature at noon?

83. Temperature Change At 10:00 in the morning in White Bear Lake, Wisconsin, Zach notices the temperature is 5 degrees below zero Fahrenheit. Write this temperature as a negative number. By noon the temperature has dropped another 10 degrees. What is the temperature at noon?

84. Snorkeling Steve is snorkeling in the ocean near his home in Maui. At one point he is 6 feet below the surface. Represent this situation with a negative number. If he descends another 6 feet, what negative number will represent his new position?

Table 2 lists various wind chill temperatures. The top row gives air temperature, while the first column gives wind speed, in miles per hour. The numbers within the table indicate how cold the weather will feel. For example, if the thermometer reads 30 °F and the wind is blowing at 15 miles per hour, the wind chill temperature is 9 °F.

Wind Chill Temperatures

	Air Temperatures (°F)							
Wind Speed	30	25	20	15	10	5	0	−5
10 mph	16	10	3	−3	−9	−15	−22	−27
15 mph	9	2	−5	−11	−18	−25	−31	−38
20 mph	4	−3	−10	−17	−24	−31	−39	−46
25 mph	1	−7	−15	−22	−29	−36	−44	−51
30 mph	−2	−10	−18	−25	−33	−41	−49	−56

Table 2

85. Wind Chill Find the wind chill temperature if the thermometer reads 25 °F and the wind is blowing at 25 miles per hour.

86. Wind Chill Find the wind chill temperature if the thermometer reads 10 °F and the wind is blowing at 25 miles per hour.

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87. Wind Chill Which will feel colder: a day with an air temperature of 10 °F and a 25-mph wind, or a day with an air temperature of −5 °F and a 10-mph wind?

88. Wind Chill Which will feel colder: a day with an air temperature of 15 °F and a 20-mph wind, or a day with an air temperature of 5 °F and a 10-mph wind?

Table 3 lists the record low temperatures for each month of the year for Lake Placid, New York. Table 4 lists the record high temperatures for the same city.

Record low temperatures for Lake Placid, New York

Month	Temperature
January	−36 °F
February	−37 °F
March	−30 °F
April	−5 °F
May	19 °F
June	22 °F
July	31 °F
August	27 °F
September	19 °F
October	11 °F
November	−11 °F
December	−31 °F

Table 3

Record high temperatures for Lake Placid, New York

Month	Temperature
January	62 °F
February	62 °F
March	78 °F
April	86 °F
May	90 °F
June	93 °F
July	97 °F
August	94 °F
September	94 °F
October	87 °F
November	74 °F
December	63 °F

Table 4

89. Temperature Figure 5 is a bar chart of the information in Table 3. Construct a scatter diagram of the same information. Then connect the dots in the scatter diagram to obtain a line graph of that same information. (Notice that we have used the numbers 1 through 12 to represent the months January through December.)

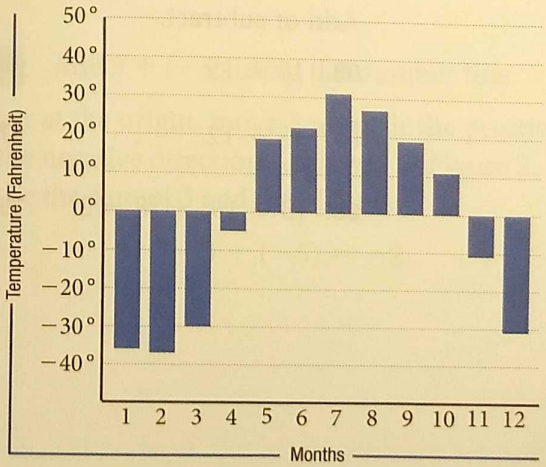


Figure 5 A bar chart of Table 3

90. Temperature Figure 6 is a bar chart of the information in Table 4. Construct a scatter diagram of the same information. Then connect the dots in the scatter diagram to obtain a line graph of that same information. (Again, we have used the numbers 1 through 12 to represent the months January through December.)

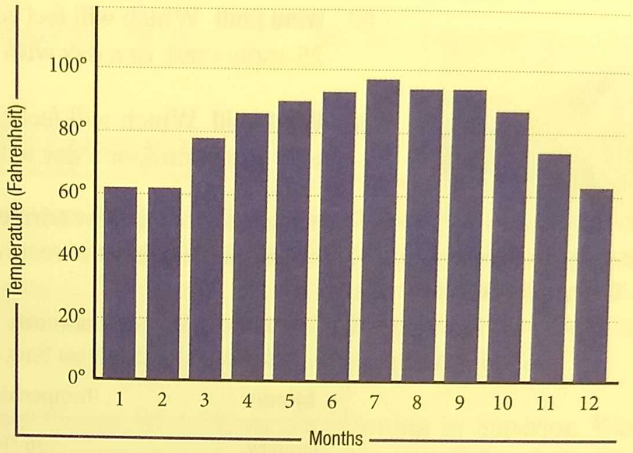


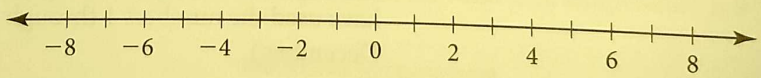
Figure 6 A bar chart of Table 4

Getting Ready for the Next Section

Problems under this heading, "Getting Ready for the Next Section," are problems that you must be able to work in order to understand the material in the next section. In this case, the problems below are variations on the type of problems you have already worked in Chapter R, along with a review of the concepts covered in Section 1.1. Together, they will prepare you for the explanations and examples in the next section.

91. Locate the number -3 on the number line. If you start from there and move 5 units to the right, where do you end up?

92. Locate the number -3 on the number line. If you start from there and move 5 units to the left, where do you end up?



Add or subtract.

93. $10 + 15$

94. $12 + 15$

95. $15 - 10$

96. $15 - 12$

97. $10 - 5 - 3 + 4$

98. $12 - 3 - 7 + 5$

99. $[3 + 10] + [8 - 2]$

100. $[2 + 12] + [7 - 5]$